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THEORETICAL AND MATHEMATICAL PHYSICS COMPETITION^(*)

– 2010 –

The Birth of a Bound State out of the Continuous Spectrum in Quantum Mechanics

Consider a one-dimensional Schrödinger equation

$$-\frac{\hbar^2}{2m}\psi_{xx} + U_0(x)\psi = E\psi,$$

where the potential U_0 is real and approaches zero at infinity:

$$U_0(x) \rightarrow 0 \quad \text{as } x \rightarrow \pm\infty.$$

Let the potential U_0 not support any bound states so that the spectrum of energies is purely continuous: $E \geq 0$. (An obvious example is a potential barrier with $U_0(x) > 0$ for all x . There are also many examples where $U_0(x)$ is not strictly positive.)

Next, let a small perturbation $\epsilon U_1(x)$ be added to U_0 so that the equation becomes

$$-\frac{\hbar^2}{2m}\psi_{xx} + U_0(x)\psi + \epsilon U_1(x)\psi = E\psi. \quad (1)$$

Here ϵ is a small parameter. Like the “parent” potential U_0 , the perturbation $U_1(x)$ is assumed to be real and vanishing at infinity:

$$U_1(x) \rightarrow 0 \quad \text{as } x \rightarrow \pm\infty.$$

Now assume that U_1 is such that the perturbed Schrödinger equation (1) does support a discrete eigenvalue, $E_0 < 0$. (Clearly, $|E_0|$ will be small if ϵ is small.) That is, the perturbation “kicks” a bound state out of the continuum.

The problem is:

- For the given “parent” potential U_0 , characterise the class of perturbations U_1 that will kick an eigenvalue out of the continuum;
- Calculate E_0 ;
- Determine whether there are any parent potentials U_0 for which no perturbation U_1 can kick out an eigenvalue.

^(*)*Terms and conditions:* Eligible to participate are UCT Science students up and including Masters Level. Contributions must be submitted by 4pm Monday, 6 December 2010 to any member of the committee: Profs. Barashenkov, Dominguez, Dunsby, Ianovsky, Peshier, Viollier. Candidates have to attest that their contributions represent own work.