Experiment E25: Decay of a resonance

E25.1 Introduction

A resonance circuit can be excited by an impulse; the free oscillations will decay away within some characteristic time.

E25.2 Bring

Bring a USB flash memory device to record the data so you can analyse it later.

E25.3 References

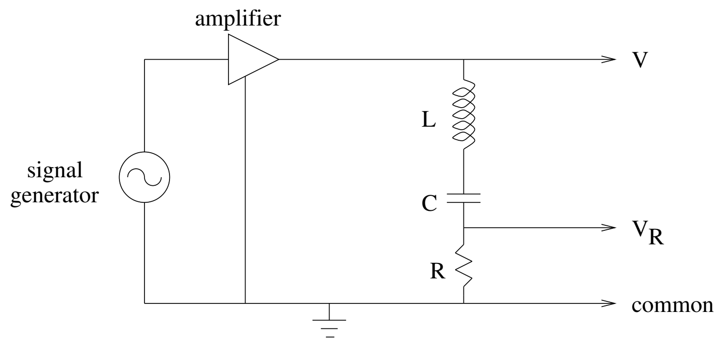
Waves and vibrations course notes.
Appendices of this manual.

E25.4 Emphasis

The purpose of this experiment is to acquaint you with measurement techniques involving the digital oscilloscope.

E25.5 Experiment

You are provided with an oscillator, an amplifier, an oscilloscope, and a box containing a capacitor and an inductor connected in series. (The resistor in the box can be used, as illustrated below, but probably introduces too much damping). You will use these to investigate free oscillations in an LCR circuit. Connect the inductor and capacitor in series across the output of the amplifier. Drive the amplifier from the signal generator with a low-frequency square wave. Use the oscilloscope to measure the voltage across the capacitor or the inductor.



Energy is stored in the capacitor and inductor while the square wave voltage is non-zero. When the square-wave voltage is zero, the circuit is effectively short circuited via the output impedance of the amplifier, and behaves as a damped harmonic oscillator as the stored energy is dissipated. (This is called "ringing", as in the note of a bell).

Set the input voltage to a square wave with a frequency of, say, 10 Hz and an amplitude of 5-10 V. You can adjust the duty cycle (i.e. on/off portion of waveform) of the wave to minimum (20%). Measure the voltage across the resistance using the digital oscilloscope, and adjust so that the decaying waveform fills the screen. You can save a file containing the digitised waveform to a USB flash drive.

E25.6 Theory

For a fuller account of the theory behind the measurement see the Waves and Vibrations lecture notes. Some important equations are summarised below. You may need to derive these to convince yourself (usually, the voltage across the capacitor is derived in books).

Assume that for $t < 0$ the square wave voltage is at V_0 (so that the charge on the capacitor is $Q = V_0 C$); at $t \geq 0$, $V_0 = 0.0$ and the low output resistance of the amplifier acts as a switch connecting (in the diagram above) the top end of the inductor to the bottom end of the resistor.

Then

$$V_R + V_C + V_L = 0 \qquad \text{so} \qquad V_R = -L\frac{dI}{dt} - \frac{Q}{C}$$

with $I = -dQ/dt$.

This linear second-order differential equation is solved for $Q(t)$ to give

$$Q(t) = CV_0 e^{-Rt/2L} \cos \omega_1 t$$

and so

$$I(t) = CV_0\omega_1 \left[\sin(\omega_1 t) + \frac{R}{2L\omega_1} \cos(\omega_1 t) \right]$$

Note that, for flexibility in fitting, it is useful to introduce a small phase shift in the phase of the sin and cos: $\sin(\omega_1 t + \phi)$, etc.

Thus the voltage V_R across the capacitor at time t after the square-wave voltage has gone to zero is

$$V_R = V_m e^{-Rt/2L} \left[\sin(\omega_1 t + \phi) + \frac{R}{2L\omega_1} \cos(\omega_1 t + \phi) \right]$$

where L is the inductance, R the effective damping resistance, and ω_1 is the angular frequency, related to the undamped angular frequency $\omega_0 = 1/\sqrt{LC}$ by

$$\omega_1^2 = \omega_0^2 - \left(\frac{R}{2L}\right)^2$$

The phase constant ϕ is introduced to allow for a small offset in the start time, and the amplitude $V_m = RCV_0\omega$ depends on the initial square wave amplitude V_0.

The voltage across the inductor can be derived from the above

$$V_L(t) = -L\frac{dI}{dt} = \ldots$$

E25.7 Measurements

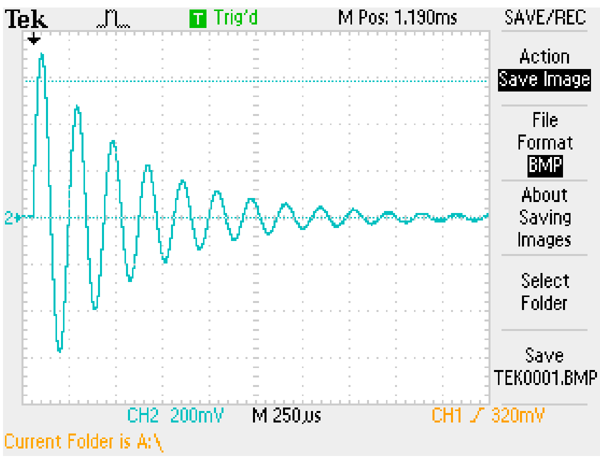
By fitting the theoretical expression to the measured curve, find R/L, the Q of the circuit and the resonance frequency. Note that R is the total resistance in the circuit: resistor (if used) + inductor + output impedance of amplifier, and so is not the tabulated value on the noticeboard. The tabulated C can be used to find L from ω_0. Hence you can find R. There will be a nice calculation of the uncertainties in these latter values.

E25.8 The digital oscilloscope

The oscilloscope used in this experiment works by sampling the input signal at some small time interval and digitising the signal for display and further manipulation.

You will notice that the knobs are not calibrated. Settings are made via programmable buttons (softbuttons) along the right hand edge of the screen, and selection of function by pressing the appropriate button.

The ranges used are displayed on the screen.



To select channel 1 or 2 (CH1 or CH2), press the button with the corresponding label ("CH1 menu" or "CH2 menu"). You can then use the associated knob to change the range, and the soft buttons to select various characteristics. In particular, you should set the input attenuation to 1X and input coupling to DC. There is similar behaviour for the horizontal (time) sweep ("Horiz menu") and the triggering selection ("trig menu").

Once you have a signal there are a number of useful options. You can select a cursor (Cursor button and soft buttons) which you can use to make measurements on the screen (cursor position and difference between cursors is displayed on the screen).

A file of the currently displayed signal can be save using the "save/recall" button. Select file output using the soft buttons. You can then save the file to a USB flash drive using the "save file" softbutton. The file is saved as "comma separated values" and can be read into a spreadsheet.

Here is how to read the file in Python.

```
import csv

# read in time and voltage values from csv file
infile=csv.reader(open("tek0003.csv", "r"))

t=[]  # define initial empty lists
y=[]

# read in values from file
nstart=0
for row in infile:
    t.append(float(row[3]))
    y.append(float(row[4]))
    # find start of ringing signal ...
    if float(row[3])<=0.0: nstart += 1  # triggered for t>0

# lists to arrays
t=array(t)
y=array(y)
```