Energy Loss of Colour Charges in Nuclear Media

S. Peigné
peigne@subatech.in2p3.fr
Subatech, Nantes, France

University of Cape Town – August 2011
LECTURE 1 – CONTEXT AND MOTIVATIONS:  
*QCD and Quark Gluon Plasma*

LECTURE 2 – COLLISIONAL ENERGY LOSS

LECTURE 3 – RADIATIVE ENERGY LOSS
LECTURE 1

CONTEXT AND MOTIVATIONS:

QCD AND QUARK GLUON PLASMA
1. QCD: a non-abelian gauge theory

reminder: matter structure

- QUARK
- GLUON
- HADRON

S. Peigné

Energy Loss of Colour Charges in Nuclear Media
Standard Model of Particle Physics

→ non-gravitational interactions

electromagnetic, weak, strong interactions

obey Quantum Mechanics + Special Relativity

⇒ Quantum Field Theories (QFT)

are all described by GAUGE THEORIES

gauge groups $SU(2) \otimes U(1) \otimes SU(3)$

electroweak strong
gauge theory?

simplest case: electromagnetic interaction

‘textbook’ construction of $\mathcal{L}_{\text{QED}}$ from gauge principle:

$$\mathcal{L}_{\text{Dirac}} = \bar{\psi}(x) (i\gamma^\mu \partial_\mu - m) \psi(x) \quad \text{(free electron)}$$

$\gamma^\mu$ = Dirac matrices; $\partial_\mu = \partial / \partial x^\mu$ \quad ($\mu = 0, 1, 2, 3$)

$\mathcal{L}_{\text{Dirac}}$ invariant under $\psi \rightarrow e^{ie\Lambda} \psi$ only if $\Lambda =$ cst.

“gauge principle”: physics must be insensitive to local transformations $\psi(x) \rightarrow e^{ie\Lambda(x)} \psi(x)$

$\Rightarrow$ modify $\mathcal{L}_{\text{Dirac}}$ by: $\partial_\mu \rightarrow D_\mu = \partial_\mu + ieA_\mu$ such that:

$$D_\mu \psi \rightarrow e^{ie\Lambda(x)} D_\mu \psi \quad \text{when} \quad \psi \rightarrow e^{ie\Lambda(x)} \psi$$

$\Rightarrow$ $A_\mu \rightarrow A_\mu - \partial_\mu \Lambda$
→ **gauge invariant** lagrangian:

\[ \mathcal{L}' = \bar{\psi} \left( i \gamma^\mu D_\mu - m \right) \psi = \bar{\psi} \left( i \gamma^\mu \partial_\mu - m \right) \psi - e \bar{\psi} \gamma^\mu \psi A_\mu \]

- one ‘gauge parameter’ \( \Lambda(x) \) → one field \( A_\mu \) ↔ photon
- gauge principle dictates form of electron-photon interaction:

\[ -e \bar{\psi} \gamma^\mu \psi A_\mu \]

- kinetic term for \( A_\mu \): \(-\frac{1}{4} F_{\mu \nu} F^{\mu \nu}\) \( (F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu) \)

\[ \Rightarrow \mathcal{L}_{\text{QED}} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \bar{\psi} \left( i \gamma^\mu D_\mu - m \right) \psi \]

- QED is remarkably successful: accuracy \( \sim 10^{-8} \)!

from *simplest* gauge invariant lagrangian built with \( \psi, A_\mu \)

⇒ validates gauge principle
strong interaction: gauge principle in \textit{color space}

- quark field $\psi_i(x)$ carries color index: $i = 1, 2, 3$
- color not observable $\Rightarrow$ physics invariant under:

$$\psi_i(x) \rightarrow U_{ij}(x) \psi_j(x); \quad U(x) = e^{ig_s \Lambda^a(x) T^a} \in SU(3)$$

$\mathcal{L}_{\text{Dirac, quark}} = \sum_{i=1}^{3} \bar{\psi}_i \left(i\gamma^\mu \partial_\mu - m\right) \psi_i$

is made invariant under $\psi(x) \rightarrow U(x) \psi(x)$ by shifting:

$$\delta_{ij} \partial_\mu \rightarrow (D_\mu)_{ij} = \delta_{ij} \partial_\mu + ig_s A^a_\mu T^a_{ij}$$

- eight parameters $\Lambda^a(x) \rightarrow$ eight fields $A^a_\mu \leftrightarrow$ gluons

$$\Rightarrow \quad \mathcal{L}_{\text{QCD}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu, a} + \bar{\psi}_i \left(i\gamma^\mu (D_\mu)_{ij} - m \delta_{ij}\right) \psi_j$$
quark-gluon interaction:

\[-g_s \bar{\psi}_i \gamma^\mu A^a_\mu T^{a}_{ij} \psi_j \rightarrow \Psi_j \rightarrow \bar{\Psi}_i\]

\[A^a_\mu \quad i g_s \gamma^\mu T^a_{ij} \quad T^a_{ij} = \text{color factor}\]

gluon carries color charge

arises from non-abelian SU(3):

\([T^a, T^b] = i f_{abc} T^c\]

\[\Rightarrow F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f_{abc} A^b_\mu A^c_\nu\]

\[\Rightarrow F^a_{\mu\nu} F^{\mu\nu,a} \text{ contains gluon self-interactions:}\]

3-gluon coupling

\[f_{abc}\]

4-gluon coupling

\[f_{abe} f_{cde} + \ldots\]
QCD is formally very similar to QED but has drastically different features!

- **color confinement** (at low temperature and density)
  fields in $\mathcal{L}_{\text{QCD}} =$ colored quarks and gluons
  $\neq$ observed particles = colorless hadrons

- **asymptotic freedom (AF):** $\alpha_s(\text{energy} \to \infty) \to 0$

(Veneziano lecture, Collège de France, 2008)
radiation off energetic charge

charge radiates if quantum state is perturbed

\[ |\Psi_i\rangle \rightarrow |\Psi_f\rangle \]

\[ |\Psi_f\rangle \neq |\Psi_i\rangle \implies \text{radiation} \]

- QED: electron perturbed if \( \theta_s \neq 0 \)

- QCD: quark perturbed even if \( \theta_s = 0 \), due to rotation of quark color induced by gluon exchange

\[ \Rightarrow \text{more radiation expected in QCD} \]
2. Running coupling: QED vs QCD

effective QED/QCD couplings are “running” with energy

heuristically: test charge $q_0$ in QED

$r \gg 1/m \Rightarrow$ charge screening
(vacuum polarization)

$\alpha_{\text{eff}} \downarrow$ when $r \uparrow$

probe distance $r$

$\leftrightarrow$ momentum transfer $Q \sim 1/r$

$\Rightarrow$ in QED: $\alpha_{\text{eff}} \uparrow$ when $Q \uparrow$
Theoretically:

\[ \Pi^{\mu\nu}(q) = \mu \cdot \frac{q}{q^2} \cdot \nu = \Pi(q^2) \left( q^2 g^{\mu\nu} - q^\mu q^\nu \right) \]

\[ \sim \frac{\alpha_0}{q^2} \longrightarrow 1 + \frac{1}{3} + \frac{1}{5} + \ldots \sim \frac{\alpha_0}{q^2(1 - \Pi(q^2))} \]

\[ \Rightarrow \alpha(q^2) = \frac{\alpha_0}{1 - \Pi(q^2)} \]

\[ \text{Π}(q^2) \text{ is UV divergent} \rightarrow \text{renormalization} \]

\[ \Rightarrow \beta(\alpha) \equiv \frac{d\alpha}{d \log Q^2} = \frac{\alpha^2}{3\pi} > 0 \]

\[ \Rightarrow \alpha(Q^2) = \frac{\alpha(Q_0^2)}{1 - \frac{\alpha(Q_0^2)}{3\pi} \log \left( \frac{Q^2}{Q_0^2} \right)} = \frac{3\pi}{\log(M_{LP}^2/Q^2)} \]

arises from specific sign of \( \Pi(q^2) \)

\[ M_{LP} \equiv Q_0 \exp \left( \frac{3\pi}{2\alpha(Q_0^2)} \right) \sim 10^{277} \text{ GeV} \Rightarrow \text{Landau pole ignored in practice} \]
in QCD: \( \beta \)-function is negative

\[
\beta(\alpha_s) \equiv \frac{d\alpha_s}{d\log Q^2} = -b\frac{\alpha_s^2}{4\pi} < 0 \quad (b = \frac{11}{3}N_c - \frac{2}{3}N_f > 0)
\]

due to gluon loop

being of opposite sign to

\[
\Rightarrow \quad \alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 + b\frac{\alpha_s(Q_0^2)}{4\pi}\log\left(\frac{Q^2}{Q_0^2}\right)} = \frac{4\pi}{b\log\left(\frac{Q^2}{\Lambda_{QCD}^2}\right)}
\]

\( \Lambda_{QCD} \equiv Q_0 \exp\left(\frac{-2\pi}{b\alpha_s(Q_0^2)}\right) \simeq 200 \text{ MeV} \)
QCD: $\alpha_s(Q) \downarrow$ when $Q \uparrow$: Asymptotic Freedom

⚠️ AF does not mean that quarks don’t interact at short distance!

- AF = direct consequence of gluon self-couplings

- $\alpha_s(Q) \ll 1 \iff Q \gg \Lambda_{QCD} \approx 200\text{ MeV}$
  \[\rightarrow\] domain of perturbative QCD (PQCD)

- $Q \sim O(\Lambda_{QCD}) \Rightarrow$ expect $\alpha_s(Q) \sim O(1)$
  \[\rightarrow\] non-perturbative domain
Breakdown of PT

\[ O(1) \]

\[ \frac{1}{137} \ldots \]

\[ \Lambda_{QCD} \approx 200 \text{MeV} \]

\[ M_Z \]

\[ M_{LP} \gg M_{Planck} \]

---

Veneziano lecture, Collège de France, 2008
AF has been undoubtedly proven experimentally.
3. Successes and surprises of PQCD

- PQCD: only *predictive* analytical tool for strong interaction
  expand observables in powers of $\alpha_s(Q) \ll 1$

- during last 30 years: PQCD successfully verified for
  - many processes and observables
  - various kinematical conditions

$\Rightarrow$ validity of $\mathcal{L}_{\text{QCD}}$ and $SU(3)$ gauge symmetry:
  - color d.o.f., $N_c = 3$
  - spin $\frac{1}{2}$ quarks and spin 1 gluons
  - gluon self-couplings, asymptotic freedom,...
PQCD treatment of high-energy processes

→ QCD Factorization Theorems

\[ d\sigma(h_1 + h_2 \rightarrow h_3 + X) = \sum_{a_i b_j} \int f_{a_1/h_1} \otimes f_{a_2/h_2} \otimes d\hat{\sigma}(a_1a_2 \rightarrow b_1b_2 \ldots) \otimes D_{b_1 \rightarrow h_3} + O \left( \left( \frac{\Lambda_{QCD}}{Q} \right)^n \right) \]

- factorization holds at leading-twist
- PDFs and FFs are universal
- higher-twist terms are negligible when \( Q \) is large
large $p_T$ jet production

(Fermilab, $\sqrt{s_{p\bar{p}}} = 1.96$ TeV)

$p\bar{p} \rightarrow \text{jet}(p_T, y) + X$

$(y = -\ln \tan \theta/2)$

NLO PQCD accurate for

$p_T > 50$ GeV $\Rightarrow \alpha_s(p_T) < 0.15$

CDF@Fermilab, hep-ex/0701051
inclusive hadron distribution inside jets

PQCD $\Rightarrow$ highly virtual *parton* decays into bunch of collimated *partons* $\rightarrow$ jet of *hadrons*

resummed PQCD techniques predict jet internal structure at the *parton level*
e.g.: MLLA *hump-backed* parton energy spectrum

\[ e^+e^- \rightarrow h^\pm + X \]
\[ \xi_p = \ln \left(\frac{1}{x}\right) \]
\[ x = \frac{E_{\text{parton}}}{E_{\text{jet}}} \]

hadron and parton distributions very similar
→ hadronization is a smooth process
→ PQCD works *too* well!

∃ many examples where PQCD agrees with data
* beyond* expected domain of validity

Dokshitzer, hep-ph/0306287
4. Quark-Gluon Plasma (QGP)

some QCD challenges (among many others):

- improve PQCD accuracy
  (QCD background in search for new physics)

- understand confinement mechanism

- understand dynamics of multiparton system before transition to hadrons → QGP
Lattice QCD at finite temperature $T$

$\Rightarrow \frac{\epsilon_{\text{nuclear}}}{T^4}$ undergoes fast jump around $T_c \sim 200$ MeV

for ideal massless gas:

$\frac{\epsilon}{T^4} = \left(\frac{\pi^2}{30}\right) d \approx 0.33 d$

$d = \text{effective nb of d.o.f.}$

lattice data $\Rightarrow$ brutal change of dof around $T_c$

Bazavov et al., 0903.4379
$T < T_c : \text{ hadron gas}$

- 1 fm
- color is confined

$T > T_c : \text{ QGP}$

- $d = 47.5$ (consistent with $N_f = 3$)

$d$ increases by $\sim 15$ when $T$ exceeds $T_c$!

- lattice results show deviations from ideal $m = 0$ gas

- however: suggest $\epsilon_{\text{QGP}} / T^4 \rightarrow \epsilon_{\text{SB}} / T^4$ when $T \rightarrow \infty$

(consistent with AF: $\alpha_s(T) \rightarrow 0$ when $T \rightarrow \infty$)
QGP studies are fundamental for

- QCD, more generally gauge theories, at finite $T$
  ($T \sim T_c \Rightarrow$ apprehend confinement mechanism)
- early Universe: in QGP phase until $t \sim 10^{-5} \text{s}$

experimental search for QGP started with relativistic heavy-ion (AA) collisions

<table>
<thead>
<tr>
<th>accelerator</th>
<th>Bevalac</th>
<th>AGS</th>
<th>SPS</th>
<th>RHIC</th>
<th>LHC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{S_{NN}}$ (GeV)</td>
<td>$\sim 2.5$</td>
<td>5</td>
<td>17.3</td>
<td>200</td>
<td>2760</td>
</tr>
</tbody>
</table>

consensus: QGP has been formed at RHIC and LHC
Bjorken picture of relativistic AA collisions (1983)

\[
\gamma = \sqrt{s_{NN}}/(2m_p) \gg 1 \Rightarrow \\
\text{pancake of thickness } \frac{2R_A}{\gamma} \\
\exists \text{ always soft partons} \\
p \sim \Lambda_{QCD} \leftrightarrow \Delta z \sim 1 \text{ fm} \\
\text{energetic nucleus} \simeq \\
\text{hard pancake + soft cloud}
\]

when \(2R_A/\gamma < 1 \text{ fm} \Leftrightarrow \)

\[\sqrt{s_{NN}} > 4m_p \frac{R_A}{1 \text{ fm}} \simeq 30 \text{ GeV} \Rightarrow \]

\( \text{collision dynamics} \)

\( \text{dominated by soft partons} \)

\( \Rightarrow \text{ multiparton interactions} \)

\( \Rightarrow \text{ thermal medium} \)
Relativistic Heavy Ion Collider (RHIC)
Large Hadron Collider (LHC)
how to ascertain QGP formation?

- *colourless hadrons* are detected $\Rightarrow$ QGP observation is indirect
- ! medium goes through various phases
- two important hints for QGP formation:
  - elliptic flow $v_2$
  - large $p_T$ jet-quenching
elliptic flow $v_2$

non-central AA $\Rightarrow$ QGP with initial spatial anisotropy

Snellings, 1102.3010

during evolution: spatial anisotropy $\downarrow$; momentum anisotropy $\uparrow$

anisotropy in $\phi$ quantified by elliptic flow $v_2$:

$$v_2 = \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle} = \langle \cos 2\phi \rangle > 0$$
sizeable $v_2$ observed at RHIC & LHC
reproduced by hydro calculations assuming:

*thermal equilibrium and small $\eta/s$*

\[
\frac{\eta}{s} \simeq 0.1 \pm 0.1(\text{th}) \pm 0.1(\text{exp})
\]

Luzum & Romatschke, 0804.4015

⇒ nearly perfect fluid = strongly coupled QGP?
LHC vs RHIC $v_2$ data

$v_2(p_T)$ for the centrality bin 40-50% from the 2- and 4-particle cumulant methods for this measurement and for Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV.

$v_2\{4\}(p_T)$ for various centralities compared to STAR measurements.

ALICE Collaboration, 1011.3914 [nucl-ex]
jet-quenching

consider large $p_T$ hadron produced at $y = 0$

hadron arises from large $p_T$

parton produced in elementary pp collision

QGP $\Rightarrow$ colored particles suffer average energy loss $\Delta E$

$$
\Rightarrow R_{AA}^h = \frac{dN_{AA}^h(p_T)}{dN_{AA}^h(p_T)\big|_{\text{no QGP}}} \sim \frac{dN_{pp}^{\text{parton}}(p_T+\Delta E)}{dN_{pp}^{\text{parton}}(p_T)} < 1
$$

(due to generic behaviour $\frac{dN_{pp}^{\text{parton}}}{dp_T^2} \sim \frac{1}{(p_T^2)^n}$)

$\Delta E$ large $\Rightarrow R_{AA}^h \ll 1 \equiv \text{“jet-quenching”}$
\[ \left. dN_{AA}^h(p_T) \right|_{\text{no QGP}}^{\text{not measured}} = \# \ dN_{pp}^h(p_T) \] measured

\# = effective number of pp collisions in one AA collision

if average over impact parameter \( b \):

for given \( b \):

\[ \# = n_{\text{coll}}(b) \]

\[ n_{\text{coll}}(b \simeq 0) \sim A \cdot A^{1/3} \]

\[ \Rightarrow R_{AA}^h(p_T, b) = \frac{dN_{AA}^h(p_T)}{n_{\text{coll}}(b) \ dN_{pp}^h(p_T)} \] not directly measured
jet-quenching at RHIC ($\sqrt{s_{NN}} = 200$ GeV)

$R_{AA}^h(p_T, b \approx 0) \ll 1$

for $5 < p_T < 20$ GeV

→ considered as clear QGP signature

$R_{AA}^\gamma \sim 1$

consistent with $\Delta E_\gamma \simeq 0$

gives confidence in evaluation of $n_{coll}(b)$
jet-quenching at LHC ($\sqrt{s_{NN}} = 2.76$ TeV)

ALICE Collaboration, 1012.1004 [nucl-ex]
How to quantify jet-quenching?

Many non-trivial “ingredients” are needed:

- geometry of AA collision
- evolution of thermal medium
- in-medium parton propagation
- QCD predictions for parton energy loss?

$\Delta E(E, M, L, T, \alpha_s)$ in QGP?

rich theoretical problem

→ see next lectures
summary of lecture 1

- theory of strong interaction = QCD
  - QCD = non-abelian gauge theory
- gluon carries color charge
- running coupling: \( \alpha_s(Q) \sim \frac{1}{\ln(Q^2/\Lambda_{QCD}^2)} \)
- asymptotic freedom:
  \[ Q \gg \Lambda_{QCD} \Rightarrow \alpha_s(Q) \ll 1 \Rightarrow \text{PQCD} \]
  - PQCD too successful!

- Quark-gluon plasma
  - *deconfinement* of quarks and gluons when \( T > T_c \sim 200 \text{ MeV} \) predicted by lattice QCD
  - QGP search and study: RHIC, LHC
  - some important signals: \( v_2 \), jet-quenching