A new line density tracking algorithm for PEPT and its application to multiple tracers

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1. Introduction

Positron emission particle tracking (PEPT) is a technique used to measure the locations of a moving tracer particle using a modified positron emission tomography (PET) scanner or gamma camera [1–4]. The tracer is labelled with a radioisotope such as 18F, 68Ga or 22Na which decays via the emission of a positron. Subsequent annihilation of the positron with an electron in the surrounding material results in the emission of two nearly back-to-back 511 keV γ-rays. Each pair of γ-rays which are detected in coincidence thus defines a virtual “line of response” (LOR) which passes through, or very near to, the centre of the tracer. Since typically thousands of such LORs are recorded every second, the location of the tracer can be repeatedly triangulated as it moves within the field of view of the scanner. PEPT has been used to study the flow of material within a wide range of industrial systems [1,3].

In an idealised PEPT measurement, only two LORs would suffice to triangulate the position of the tracer, provided those two lines passed exactly through the centre of the tracer. However, the recorded data are affected by the finite size of the tracer and detectors, the distance the positron travels before decay, Compton scattering of the γ-rays between the tracer and the detectors and uncorrelated coincidences. Consequently the LORs do not all pass through the centre of the tracer, and thus a statistical approach is required to infer the best approximation of the tracer’s location. The uncertainties in these locations are most affected by the number of LORs recorded (a function of both the activity of the tracer and the type and amount of scattering material in the system) and the speed of the tracer. In some PEPT experiments, uncertainties smaller than 1 mm have been reported [5,6].

The most prominent PEPT algorithm to date is that developed at the University of Birmingham [1] which deals with the series of chronologically recorded LORs in equal groups of N events. The current implementation of the algorithm requires the user to specify N, the minimum fraction f of LORs used to triangulate each location, and a maximum acceptable error for the final solution. The algorithm then determines the point in space which minimises the sum of the distances to the LORs used, after sequentially disregarding LORs up to the limit set by f. The choice of f represents the expected fraction of uncorrupted LORs in the data set, and since this is dependent on the physical attributes of the system, multiple control experiments may be required to determine the optimal values for f and N, as was performed in Ref. [6].

While in most PEPT experiments it is sufficient to track only one tracer particle, there are many applications which could be enhanced by the simultaneous tracking of more than one tracer, such as the motion of rigid or deforming surfaces within optically opaque systems. The Birmingham algorithm is not easily adapted to this problem. One attempt at multiple particle tracking used tracers of distinctly different activity [7]. The Birmingham algorithm was modified to first isolate the most active tracer and triangulate its position, then remove the data associated with it before repeating the process for the second tracer, and so on [7,8]. The tracking of multiple particles is thus reduced to locating a single particle at a time. Although this approach was used to...
successfully track three tracers, it is limited by the requirement for each tracer to have sufficiently distinct activity from the others. More importantly, the need to use tracers with increasing levels of activity has consequences on the saturation of the data acquisition system, which puts a limit on the number of tracers that can be used at a time.

A new algorithm for analysis of PEPT data has been developed and its use is illustrated with data collected both at the Positron Imaging Centre (University of Birmingham, UK) and PEPT Cape Town (situated at iThemba LABS, South Africa). The algorithm has been successfully applied to the tracking of up to 16 tracers, and in principle is not limited in this regard.

2. The line density algorithm

The method is based upon the premise that the location of the tracer is within a region of space through which the highest density of lines of response pass. The algorithm requires a Cartesian 3-dimensional grid to be specified within the field-of-view (FOV) of the scanner. The resolution of this mesh is in principle not restricted, except by the consequences on computing resources. Considering a particular time slice of the data (usually a few milliseconds and thus only including several hundred LORs) the algorithm determines the voxels of the 3D mesh through which each LOR passes and increments a counter on each of these voxels (an approach more commonly known as “backprojection”). The resulting count matrix thus represents a distribution of the density of the LORs over the FOV of the scanner. This approach is illustrated in Fig. 1 for 50 LORs over a coarse mesh. The voxel containing the highest count and hence the source of the $\gamma$-rays is clearly identified. After identifying the peak in the density matrix, three slices through this peak, one voxel width thick and parallel to each of the three dimensions, are considered. Fig. 2 shows the distribution of counts along these three slices for 4 ms of a typical data set. Gaussian fits are then applied to these distributions, which is reasonable provided that the size of the tracer is smaller than dimension of the voxels used. The best approximation of the location of the tracer is given by the centroids of these three Gaussian fits. The uncertainty in the location is proportional to the full width at half maximum (FWHM) of these Gaussians, and inversely proportional to the square root of the contributing LORs. The time step used to slice up the data set is specified, which determines the approximate number of lines of response used per location. By repeatedly applying this method the chronological locations of the tracer may be realised.

Fig. 1. Field of view (xy-plane shown) of a PET scanner is meshed into 3-dimensional voxels, of dimension 55 × 55 mm². The location of the tracer is within the voxel containing the most overlapping LORs, which is also the voxel having the most number of LORs passing through it.

Fig. 2. Distributions of voxel counts for about 500 lines of response from a typical data set, with their Gaussian fits. The centroids of the fitted Gaussians provide best estimates of the ($x, y, z$) coordinates of the tracer.
When more than one tracer is being tracked then the algorithm requires that the approximate initial positions of the tracers be specified. The simplest way of providing this information is by defining regions within the field of view (FOV) of the camera where only one tracer is present at the start of the run, which is readily achieved by visual inspection of the first time slice of the data. The algorithm then isolates all LORs in each of these regions and determines the location of the tracer in each region. For subsequent iterations the algorithm uses the previous locations of the tracer to estimate which LORs should be used to calculate the next location. This is achieved through extrapolation: the previous two locations of the tracer are used to approximate the next location. This is achieved through extrapolation: the previous two locations of the tracer are used to approximate the next location. This is achieved through extrapolation: the previous two locations of the tracer are used to approximate the next location. This is achieved through extrapolation: the previous two locations of the tracer are used to approximate the next location.

The size of each tracer was about 2 mm and was labelled with $^{18}$F, $^{68}$Ga or $^{22}$Na. This resulted in a mean spatial resolution of 4.8 ± 0.2 mm full width at half maximum (transaxial, 1 cm off-axis) and 5.6 ± 0.5 mm (axial, on-axis) [10]. A special mechanical apparatus was constructed to move multiple tracers along predetermined oscillatory paths in one or two dimensions. This ensured that their motion was known a priori and thus the output of the tracking algorithm could be verified. For most runs the tracers moved at a maximum speed of about 0.2 m s$^{-1}$, although higher speeds were also tested. The size of each tracer was about 2 mm and was labelled with between 50 $\mu$Ci and 300 $\mu$Ci of $^{18}$F, $^{68}$Ga or $^{22}$Na. This resulted in between 20 000 and 60 000 LORs recorded per second per tracer.

Fig. 3 shows a plot of position versus time for a single tracer particle moving in a circular motion, as measured with the ADAC Forte gamma camera. The particle was tracked using the new line density algorithm (applied with a time step of 25 ms), shown by the large black dots, and the Birmingham algorithm (applied with $f=0.40$ and $N=320$), shown by the smaller grey dots.

3. Experimental validation

Experiments were conducted using both the ADAC Forte gamma camera at the Positron Imaging Centre at the University of Birmingham, UK [1], and the Siemens ECAT HR+ scanner at the laboratories of PEPT Cape Town, situated at the South Africa. The ADAC Forte is a dual head camera with an intrinsic spatial resolution of about 8 mm [2]; while the HR+ scanner has a ring configuration with an intrinsic mean spatial resolution of 4.8 ± 0.2 mm full width at half maximum (transaxial, 1 cm off-axis) and 5.6 ± 0.5 mm (axial, on-axis) [10]. A special mechanical apparatus was constructed to move multiple tracers along predetermined oscillatory paths in one or two dimensions. This ensured that their motion was known a priori and thus the output of the tracking algorithm could be verified. For most runs the tracers moved at a maximum speed of about 0.2 m s$^{-1}$, although higher speeds were also tested. The size of each tracer was about 2 mm and was labelled with between 50 $\mu$Ci and 300 $\mu$Ci of $^{18}$F, $^{68}$Ga or $^{22}$Na. This resulted in between 20 000 and 60 000 LORs recorded per second per tracer.

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$$f(t) = a \sin(b(t-c)) + d,$$
was fitted to the \(y\) and \(z\) data points, and the results of the fit are shown in Table 1. The fit variables for the line density and Birmingham algorithms agree well with each other.

Data from a run using a single stationary tracer positioned at the centre of the field of view of the HR\(^{++}\) scanner were analysed using both the Birmingham algorithm and the present algorithm. The Birmingham algorithm had an average time step of 1.0 ms (using \(f=0.40\) and \(N=210\)), and the line density algorithm had an average time step of 1.0 ms, using a mesh size of 2 mm. The mean and standard deviation of the 5000 measured locations over 5 s were (8.69, 10.70, \(-4.42\)) \(\pm\) (1.40, 1.13, 0.72) mm using the Birmingham algorithm and (6.66, 11.29, \(-4.02\)) \(\pm\) (1.37, 1.40, 0.70) mm using the line density algorithm.

Fig. 5 shows the results from tracking of eight tracers simultaneously using the ADAC Forte camera, with a mesh voxel volume of \(2 \times 2 \times 2\) mm\(^3\) and a time step of 150 ms. The eight tracers were held separate from each other and moved in an oscillatory fashion in one dimension. The measured coordinates of three of these tracers are also shown.

Fig. 6 shows the results from tracking of 16 tracers simultaneously using the HR\(^{++}\) scanner, using a mesh voxel dimension of \(2 \times 2 \times 2\) mm\(^3\) and a time step of 25 ms (but using only 2 ms of data per location). The 16 tracers

Table 1

<table>
<thead>
<tr>
<th>(y) (z) (y) (z)</th>
<th>Birmingham</th>
<th>Line density</th>
<th>Birmingham</th>
<th>Line density</th>
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</thead>
<tbody>
<tr>
<td>(a) (mm)</td>
<td>54.01 (\pm) 0.04</td>
<td>54.03 (\pm) 0.08</td>
<td>53.00 (\pm) 0.11</td>
<td>51.98 (\pm) 0.23</td>
</tr>
<tr>
<td>(b) (\text{s}^{-1})</td>
<td>2.913 (\pm) 0.001</td>
<td>2.913 (\pm) 0.001</td>
<td>2.913 (\pm) 0.002</td>
<td>2.913 (\pm) 0.003</td>
</tr>
<tr>
<td>(c) (ms)</td>
<td>784.6 (\pm) 0.4</td>
<td>784.5 (\pm) 0.8</td>
<td>1323 (\pm) 1</td>
<td>1324 (\pm) 2</td>
</tr>
<tr>
<td>(d) (mm)</td>
<td>308.2 (\pm) 0.1</td>
<td>312.2 (\pm) 0.1</td>
<td>403.6 (\pm) 0.1</td>
<td>403.4 (\pm) 0.2</td>
</tr>
<tr>
<td>(R^2) value</td>
<td>0.9999</td>
<td>0.9997</td>
<td>0.9994</td>
<td>0.9968</td>
</tr>
</tbody>
</table>
were rotated at fixed radii at a constant frequency of 0.95 Hz. Also shown are the measured coordinates of three of these tracers over time. Fig. 7 shows results from data measured using four freely moving tracers, tumbling randomly within a closed container of volume 2 dm³. These tracers were moving independently and could thus collide with or pass very near to each other. Although the algorithm is not able to maintain unique identification of colliding tracers after such an event, this can be fixed by manual intervention.

4. Discussion

The primary parameters in the algorithm are the resolution of the mesh that is generated over the field-of-view of the camera, and the time step used to slice the data, which sets the approximate number of LORs used for each location. Fig. 8 shows the FWHM of the fitted Gaussian as a function of mesh dimension for data from a typical run using a single tracer. The FWHM decreases smoothly with a decrease in the mesh voxel size, approaching an asymptotic value as the voxel dimension approaches zero; this asymptotic behaviour could not be fully realised because of computer processing limitations. Fig. 9 shows the FWHM of the fitted Gaussian as a function of the number of LORs used. For this particular run, the optimum number of LORs is seen to be around 100: a larger number does not decrease the FWHM, and a smaller number produces an erratic value for the FWHM. This behaviour of the parameters appears to be reasonably consistent across the experiments performed to date, and can thus be regarded as approximately optimised for each new experiment. Therefore the line density algorithm has the advantage of requiring very few, if any, control experiments to be conducted for the purposes of parameter optimisation.

Separate runs were also conducted with single tracers in the same positions and undergoing the same motion as each tracer in the 8 particle system discussed above (Fig. 5). Fig. 10 shows the average FWHM for the fitted Gaussians (z-coordinate) for both the 8 tracer system and the 8 tracers tracked individually. The FWHM for the system of 8 tracers is systematically larger than the single particle case, but never by more than 12%. This suggests that the new algorithm essentially is able to treat each tracer in a multiple particle system as if it was the only tracer in the system. This
slight increase in the FWHM for the 8 particle system is due to the fact that there is more scatter and noise when there are more particles present in the field-of-view. The FWHM also increases towards the edges of the field of view of the camera as would be expected.

The FWHM of the fitted Gaussians is proportional to the statistical uncertainty in the location of the tracer, which may be estimated by \( \text{FWHM} / \sqrt{N} \) where \( N \) is the total number of LORs in that time slice. There are many factors which contribute non-linearly to the final uncertainty in the location of the tracer, such as the finite size of the detector cells in the scanner, the range of the emitted positrons (\( \sim 0.2 \text{ mm for } ^{18}\text{F} \)), the non-linearity of the \( \gamma \)-pair (up to 0.5\(^\circ\)), as well as the uncertainty associated with using a finite mesh size during reconstruction (\( \sim 2 \text{ mm} \)).

Any multiple particle tracking algorithm is susceptible to the fact that if two particles come too close together then it would not be possible to identify the LORs associated with each individual tracer. The resolving power of the scanner and the algorithm is a measure of how close two particles can get before the algorithm fails to distinguish between them. This is affected by the sensitivity of the scanner, the quality of the data, and more importantly in this context the accuracy of the algorithm. Data measured with the HR++ scanner was used to test the resolving power of the line density algorithm. This was determined by tracking two particles approaching each other until they came into contact; the particle separation in the iteration immediately before that in which only a single particle could be located was taken as representing the resolving power. The resolving power is dependent on the dimensions of the cube, or region-of-interest, used to isolate the LORs associated with a particular particle, since if two particles fall within the same cube then the algorithm will only locate one of them. Using a cube volume of \( 10 \times 10 \times 10 \text{ mm}^3 \) a resolving power of 9.04 mm was achieved. Even though at this separation the second particle is not enclosed within the cube of the first, a sufficient number of LORs associated with the second particle could pass through this cube and cause the algorithm to locate only the second particle. For most analyses the average resolving power is observed to be approximately 20 mm. If the algorithm were adapted to handle these collisions then a “swopping” of the tracers after a collision or near miss may occur, in some applications this “swopping” may not matter, while in others the tracers may need to be constrained in ways that ensure that these collisions or near misses do not occur. The possible application of this algorithm to allow for interactions between tracers is yet to be explored.

5. Conclusion

A new tracking algorithm for PEPT has been developed based on the meshing of the volume defined by the field of view of the scanner. This new technique has been shown to provide very similar results to the Birmingham algorithm in the case of a single tracer, and has been used to simultaneously track up to 16 tracers, although in principle this is not a practical limit. The uncertainties in the measured locations are influenced mainly by the dimension of the mesh voxels used, and the number of LORs available in each time slice. An important difference between this approach and the triangulation method of the Birmingham algorithm is that the entire ensemble of LORs is used without prejudice. Since the “unwanted” LORs (associated with Compton scattering and false coincidences) are evenly spread throughout the mesh voxels, this provides a direct measure of the degree of scattering in the system. The parameter \( f \) in the Birmingham algorithm can be thought of as an estimate of the scatter in the system, but must be optimised for each run. The choice of mesh size and time slice in the present algorithm are easily optimised and transferred across runs, thus requiring very few, if any, control experiments to be conducted for the purposes of parameter optimisation, and they translate to a measure of the uncertainties in the locations in a more natural way. The technique offers interesting possibilities for new approaches to studying systems of flow via PEPT, and the tracking of the motion of rigid or deforming surfaces within optically opaque systems.

The extension of this line density algorithm from the tracking of a few tracers to a very large number provides a novel approach to (time dependent) PET image construction. Traditional PET imaging techniques typically rely on static or quasi-static systems. The present approach offers a method of dynamic imaging of systems of flow, currently under development, without an a priori restriction on the time slicing of the data, and without the need for sinograms.

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References