University of Cape Town - Department of Physics
Honours Computational Physics

TOPIC 1 - BASIC MONTE CARLO : SAMPLING - TRANSFORMATION METHOD

This worksheet accompanies the EJS simulation `BasicMC_No5_TransformationMethod.jar`

The challenge: We need to sample according to a distribution $p(y)$ with $y \in [a, b]$ but have access just to a uniform random generator for x over the interval $[0, 1]$.

One option is to consider the cumulative distribution function of $p(y)$:

$$F(y) = \int_a^y p(y')\, dy'.$$

We identify $F(y)$ with x, perform the integration and invert the result to obtain a transformation law $y(x)$. The so-called **'transformation method'** then proceeds as follows:

- Use the uniform random generator to return a random x_i in the interval $[0, 1]$
- Use the generated x_i to find y_i, using the derived transformation law $y(x)$
- Repeat until enough samples are collected
- The set $\{y_i\}$ then follows the distribution $p(y)$

The greatest advantage of this method is that no work is wasted; every sampled x_i leads to an accepted y_i. Infinite domains are also no problem, provided the distribution can be analytically integrated and the result inverted. One disadvantage, however, is that the sample distribution $p(y)$ has to be normalised.

Questions:

1. Use the transformation method to sample according to the following probability distribution functions (check your results using the associated EJS simulation):

    i) $p(y) = \frac{1}{2}$ with $y \in [2, 4]$;
    ii) $p(y) = \cos(2y)$ defined on the region $-\pi/4 \leq y \leq \pi/4$;
    iii) $p(y) = \frac{1}{\sqrt{8}-\sqrt{3}} \frac{y}{\sqrt{y^2-1}}$ defined on the region $2 \leq y \leq 3$;
    iv) $p(y) = \frac{1}{\sqrt{8}} \frac{y}{\sqrt{y^2-1}}$ defined on the region $1 < y \leq 3$;
    v) $p(y) = e^{-y}$ with $y \in [0, \infty)$.