

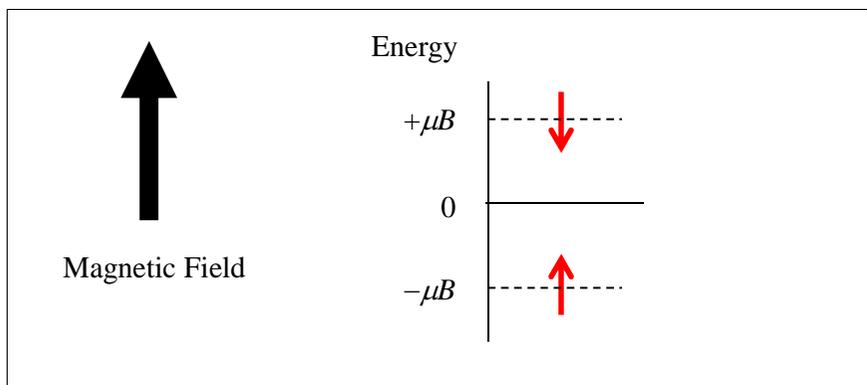
Guided Inquiry Worksheet 3:

Two-State Paramagnet

The two-state paramagnet model is another simple model that can be used to investigate the fundamental ideas of Statistical and Thermal Physics.

Within this model, we consider N non-interacting magnetic dipoles, with dipole moment μ , in an external magnetic field of strength B . Each dipole can point either in the direction of the magnetic field, or in the opposite direction (i.e. the dipoles are “two-state”). A dipole pointing in the direction of the magnetic field has less energy than one pointing in the opposite direction, since it requires energy to flip a dipole from being aligned with the field to being anti-aligned.

Assuming that the external magnetic field points upwards, the energy states of the dipole are $-\mu B$ and $+\mu B$ for up (\uparrow) and down (\downarrow) dipoles respectively:



Activity 1: Single Small Two-State Paramagnet

Consider a collection of $N = 3$ *non-interacting dipoles* in an external magnetic field that is pointing upwards

1.1) List all of the possible microstates of this system. The simplest way to represent a microstate within this model is to use arrows (e.g. $\downarrow\downarrow\uparrow$).

MICROSTATES:

1.2) There are several ways to describe macrostates of the system. List all that you can think of.

Pick one of these descriptions and, for each macrostate, list its member microstates:

Macrostate	Microstates

One way to describe a macrostate for this system is to specify the number of “up dipoles”. Alternatively, one could specify the number of “down dipoles”, the energy of the system or the total magnetic moment of the system M (specifying one of these properties fixes the other three).

1.3) Using the number of “up dipoles” to characterise macrostates, complete the table below for the $N = 3$ dipole system.

N_{\uparrow}	Energy U (units of μB)	Multiplicity Ω	Entropy S (units of k)

Activity 2: Medium Sized Two-State Paramagnet

Consider a collection of $N = 100$ **non-interacting dipoles** in an external magnetic field that is pointing upwards

Two-state paramagnet microstates are easily counted using one of the basic combinatorics examples covered in class. Convince yourself that the multiplicity of the macrostate with N_{\uparrow} of the N dipoles pointing up is,

$$\Omega(N, N_{\uparrow}) = \frac{N!}{N_{\uparrow}!(N - N_{\uparrow})!}$$

Once the multiplicity of a macrostate is known, its entropy is easily found using

$$S = k \ln \Omega(N, N_{\uparrow})$$

The temperature of the system in a given macrostate is then determined using

$$T = \left(\frac{\partial S}{\partial U} \right)^{-1}$$

and its heat capacity (at fixed volume) by

$$C = \frac{\partial U}{\partial T}$$

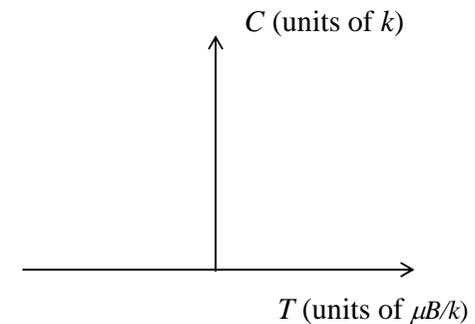
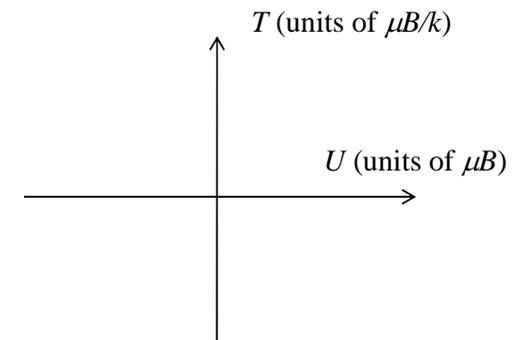
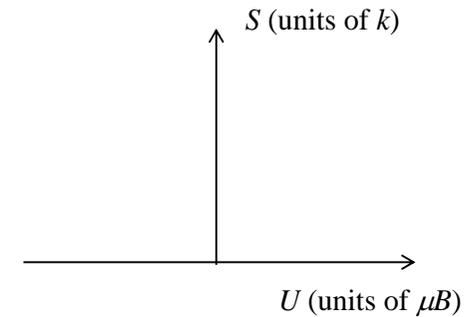
A “centred-difference” finite difference approximation can be used to evaluate both temperature and heat capacity.

Using these results, complete the spreadsheet provided for the system of 100 dipoles.

2.1) How many macrostates are available to this system?

2.2) You should have noticed that the temperature of the system is negative for some macrostates. Is this physical? Can you identify what property (or properties) of this model is (are) responsible for this behaviour?

2.3) Draw the following sketch graphs for the system:



Activity 3: Applet - Two-State Paramagnet

*Use the EJS applet, **No3_ParamagnetMultProbEntropy.jar**, to investigate the properties of a collection of non-interacting dipoles in an external magnetic field that is pointing upwards*

3.1) Firstly, confirm your earlier results in **(1.3)**.
Do you notice any discrepancies?

3.2) Now, investigate the system considered in Activity 2.
Are your spreadsheet calculations confirmed?

Take note of where certain macrostate quantities are undefined.
Why are they undefined?

Did you list them as undefined?

3.3) Compare your sketch graphs of **(2.3)** with those of the applet.
Do they agree? Pay special attention to concavity, end points, maxima, minima, etc.

S:

T:

C:

3.4) The simulation allows you to adjust the number of dipoles N in the system, as well as the number of dipoles pointing down.

What effect does changing the number of down dipoles have on each part of the display?

Make sure you understand exactly what is shown in the pink panel and that you understand the relationships between the various plots.

Notes:

3.5) What behaviour do you observe for the heat capacity about $T = 0$?

Does this behaviour agree with the equipartition theorem? Explain.

Why must the observed behaviour be so?