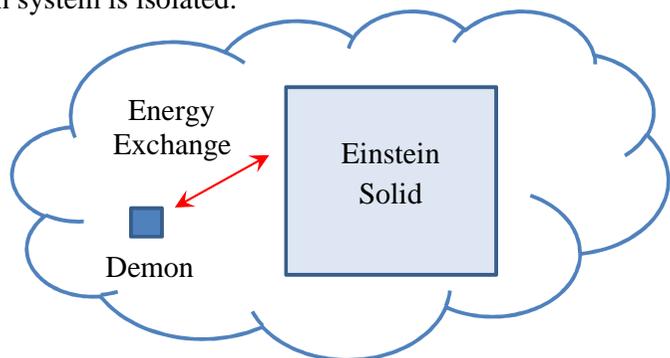


Guided Inquiry Worksheet 5:

Einstein Solid Temperature Demon

In this activity, we will consider the case of an Einstein solid in contact with a temperature demon (a single oscillator thermometer that exchanges energy with the Einstein solid). The combined solid-demon system is isolated.



All thermometers require energy exchange with the object whose temperature is sought. The temperature of the object affects the amount of energy transferred to (or from) the thermometer. This affects some property of the thermometer, which is used to infer the thermometer's temperature. This then serves as a measure of the object's temperature. As an example, a mercury thermometer relies on the expansion of a column of mercury as energy is added (its temperature is increased).

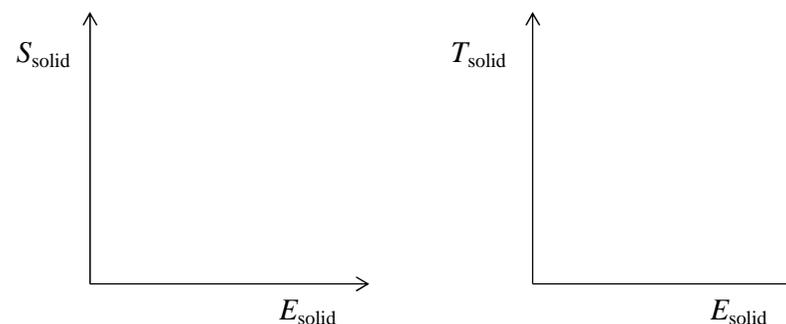
In the case of our thermometer (the temperature demon), its energy distribution is used as a measure of temperature. Everyday thermometers are large in size (of order Avogadro's number of particles). Our temperature demon, on the other hand, is comprised of a single oscillator, but it highlights important features of temperature measurement.

Activity 1: Small Einstein Solid & Demon

Consider an Einstein solid with 256 oscillators initially containing 100 energy units. This solid is then put in thermal contact with a single oscillator (a temperature demon).

1.1) How many energy macrostates are available to the Einstein solid?

1.2) Draw sketch graphs of the entropy and temperature of the energy macrostates of the Einstein solid.

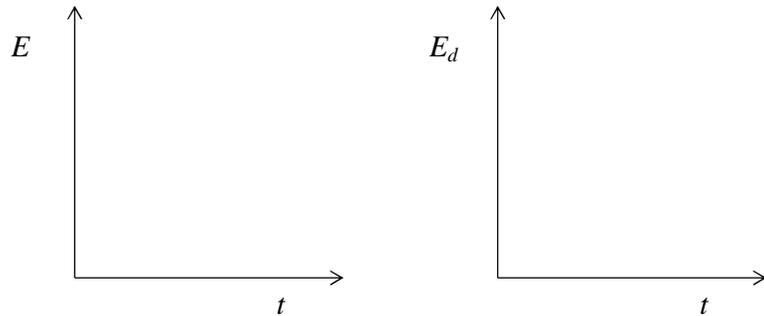


1.3) On the temperature graph just drawn, indicate the temperature that our demon wishes to measure.

You should have sketched a temperature graph that increases monotonically with energy. As such, the more energy the Einstein solid contains, the higher its temperature.

1.4) Describe what happens as the Einstein solid and demon interact (assume that the demon initially contains no energy).

i) Plot the Einstein solid and demon energies (E and E_d respectively) versus time.



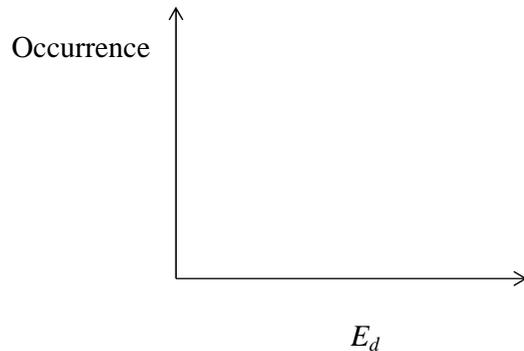
ii) How would you characterise equilibrium in this system?

1.5) What (if any) are the implications of your answer to 1.4 for the temperature measurement?

What effect do you think does the Einstein solid size have on the temperature measurement? Explain.

1.6) Now consider the temperature demon. Sketch its energy distribution (i.e. the demon energy histogram).

Hint: Are all demon energies equally likely?



How is the energy distribution just drawn affected by the initial energy (temperature) of the Einstein solid?

Based on this, how does the demon energy distribution infer the temperature of the Einstein solid?

Consider, for the moment, a temperature demon in contact with a large Einstein solid (of order Avogadro's number of oscillators) with many units of energy. The solid is so large that its temperature T is unaffected by energy exchange with the demon. For such a situation, the demon's energy distribution is given by a Boltzmann factor (the large Einstein solid acts as a heat bath of fixed temperature T):

$$P(E_d) \propto e^{-E_d/kT}.$$

It is convenient to plot the logarithm of this probability, since the temperature is related to the slope of $\ln P$. In fact, the temperature is the inverse slope (up to a factor of $-k$). This then presents a way of inferring the temperature of large systems: put the system in thermal contact with a much smaller system and measure the inverse slope of the logarithm of the smaller system's energy distribution.

Now, consider the EJS simulation launched by running the file `No5_EinsteinSolidTemperatureDemon.jar`.

- The Einstein solid – demon system is isolated. The number of energy units in this system can be adjusted by editing the field in the main display. All of this energy is originally in the Einstein solid, but after interaction starts it is shared between the demon and the Einstein solid.
- The size of the Einstein solid can be adjusted using the slider in the main display. The demon always contains just a single oscillator.
- Demon and Einstein solid properties can be displayed by checking the appropriate boxes.
- In this simulation, colour is an indication of energy content in an oscillator.

1.7) Use the simulation to confirm your earlier responses. Here is a checklist together with space to comment on discrepancies:

- Solid entropy and temperature sketch graphs (1.2)

- Energy evolution and equilibrium (1.4)

- Demon energy distribution (1.6)

1.8) Using the data tools provided, determine the inverse slope of the $\ln P(E_d)$ graph and, hence, obtain an estimate of the temperature of the Einstein solid (wait for sufficient samples to be generated).

(At high energies, low statistics may warrant excluding these data points from the fit; simply select those points in the data table that you want to include in the fit.)

How does this temperature estimate compare to the actual temperature of the Einstein solid that is sought? Explain.

The simulation contains data tools that allow best-fits to data and various other plotting options. Right click inside any of the plot panels and select

Elements options->TraceDemonProbability->Analyze data

This will allow you to analyse the $\ln P(E_d)$ data. You can follow a similar procedure to analyse data in any of the other plotting panels (e.g. the Temperature data can be analysed by selecting

Elements options->TempTrace->Analyze data

1.9) Consider an Einstein solid with 256 oscillators and 200 units of energy. How good an indication is the inverse slope of the $\ln P(E_d)$ graph of the Einstein solid temperature in this case? Explain.

(At high energies, low statistics may warrant excluding these data points from the fit; simply select those points in the data table that you want to include in the fit.)

1.10) Consider an Einstein solid with 4 oscillators and 100 units of energy. How good an indication is the inverse slope of the $\ln P(E_d)$ graph of the Einstein solid temperature in this case? Explain.

1.11) The $\ln P(E_d)$ plot includes a straight black line. What does this straight line represent? Is it the best fit to the data? How is the slope of this line related to the Einstein solid temperature?

1.12) Summarise the major findings from this activity.