NLO corrections to the dipole factorization of F2 and FL at low x

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Exploring QCD frontiers: from RHIC and LHC to EIC

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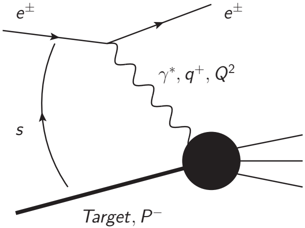
Outline

1. Introduction: Dipole factorization of DIS at LO
2. NLO corrections to the Dipole factorization of DIS
   G.B. arXiv:1112.4501 [hep-ph], *accepted in* PRD
   - Photon wave-functions at NLO
   - NLO virtual photon cross sections
   - Some remarks about kinematics of parton cascades in mixed space
3. Improving the treatment of kinematics in the BK equation
   G.B., *in preparation*

Deep inelastic Scattering (DIS) structure functions



$$\frac{\mathrm{d}^2\,\sigma^{DIS}}{\mathrm{d}x\,\mathrm{d}Q^2} = \frac{4\pi\,\alpha_{em}^2}{x\,Q^4}\left\{\left(1-y+\frac{y^2}{2}\right)\,F_2(x,Q^2) - \frac{y^2}{2}\,F_L(x,Q^2)\right\}$$

$$x = \frac{Q^2}{2(q\cdot P)} \simeq \frac{Q^2}{2q^+\,P^-} \quad \text{and} \quad y = \frac{Q^2}{xs}$$

Virtual photon cross sections

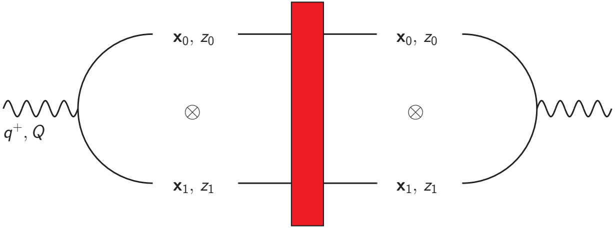
$$F_2(x, Q^2) \equiv F_T(x, Q^2) + F_L(x, Q^2)$$

$$F_{T,L}(x, Q^2) = \frac{Q^2}{(2\pi)^2\, \alpha_{em}}\, \sigma^{\gamma}_{T,L}(x, Q^2)$$

σ^{γ}_{T} and σ^{γ}_{L} : Transverse and longitudinal virtual photon - target total cross sections.

$$\frac{\mathrm{d}^2\, \sigma^{DIS}}{\mathrm{d}x\, \mathrm{d}Q^2} = \frac{\alpha_{em}}{\pi\, x\, Q^2} \left\{ \left(1 - y + \frac{y^2}{2}\right) \sigma^{\gamma}_{T}(x, Q^2) + (1-y)\, \sigma^{\gamma}_{L}(x, Q^2) \right\}$$

Dipole factorization of DIS at LO order



$$\sigma^{\gamma}_{T,L}(x,Q^2) \quad = \quad \frac{2N_c\,\alpha_{em}}{(2\pi)^2}\sum_f e_f^2 \int \mathrm{d}^2\mathbf{x}_{01} \int_0^1 \mathrm{d}z_1\ \ \mathcal{I}^{LO}_{T,L}(x_{01},1-z_1,z_1)\ \ \sigma_{dipole}(x_{01},\cdots)$$

Nikolaev, Zakharov (1991)

$\mathcal{I}^{LO}_{T,L} \propto |\text{virtual photon light-front wave-function}|^2$

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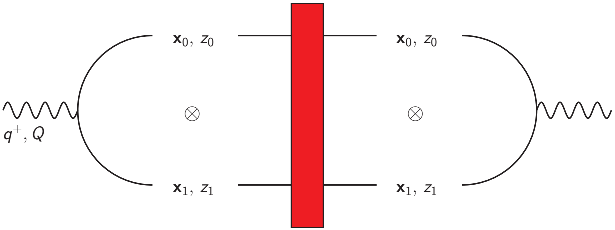

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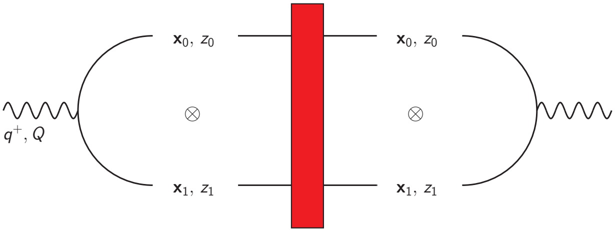
Nikolaev, Zakharov (1991)

$$\mathcal{I}^{LO}_{L}(x_{01},z_0,z_1) \quad = \quad 4Q^2 z_0^2 z_1^2\, \mathrm{K}_0^2\Big(Q\sqrt{z_0 z_1 x_{01}^2}\Big)$$

Dipole factorization of DIS at LO order



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Nikolaev, Zakharov (1991)

$$\mathcal{I}^{LO}_T(x_{01},z_0,z_1) = \left[z_0^2+z_1^2\right] z_0 z_1\, Q^2 \mathrm{K}_1^2\left(Q\sqrt{z_0 z_1 x_{01}^2}\right)$$

Dipole cross section

Optical theorem:

$$\sigma_{dipole}(x_{01}, \cdots) = 2 \int \mathrm{d}^2\mathbf{b} \left[1 - \langle \mathcal{S}_{01} \rangle_{\ldots}\right]$$

Impact parameter: $\mathbf{b} = (\mathbf{x}_0 + \mathbf{x}_1)/2$

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$\mathcal{S}_{01}$: dipole-target elastic S-matrix:

$$\mathcal{S}_{01} \quad = \quad \frac{1}{N_c}\,\mathrm{tr}\Big(U(\mathbf{x}_0)\;U^\dagger(\mathbf{x}_1)\Big)$$

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Fondamental Wilson line in the semiclassical gluon field of the target:

$$U(\mathbf{x}) \quad = \quad \mathcal{P}\;\exp\Big[ig\int \mathrm{d}x^{+}\;T^{a}\,\mathcal{A}_a^{-}(x^{+},\mathbf{x},0)\Big]$$

$\langle\cdots\rangle_{...}$: average over the target state (from Color Glass Condensate formalism), with some high-energy factorization scheme & scale.

Phenomenological studies

In practice, Dipole cross section or S-matrix taken from:

- Phenomenological models
- B-JIMWLK or BK or BFKL evolution, + initial conditions

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State of the art: fits of F_2 data with numerical simulations of BK with running coupling inserted in the LO dipole factorization:
Albacete, Armesto, Milhano, Quiroga, Salgado (2011)
Kuokkanen, Rummukainen, Weigert (2011)
Heavy quark production or diffractive structure functions also included in the fits, and comparison with F_L data is provided.

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Aim: precision predictions for EIC and LHeC
$\Rightarrow$ use the NLO dipole factorization formula with numerical solutions of the NLL BK equation of
Balitsky, Chirilli (2008)

Calculations of NLO impact factors for DIS at low x

- NLO photon impact factor for the BFKL formalism in momentum space
  Bartels et al., and Fadin et al. (2000-2005)

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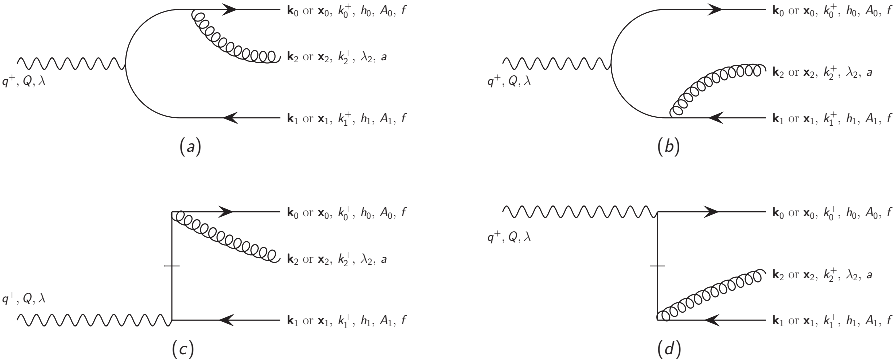
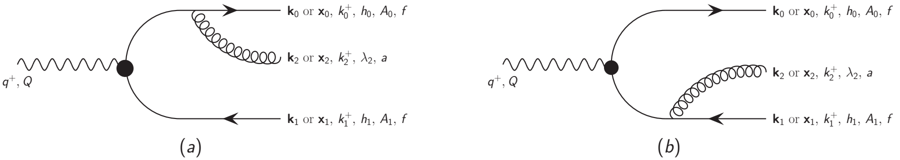
- NLO dipole factorization formulae for σ_T^γ and σ_L^γ
  G.B. arXiv:1112.4501 [hep-ph]
  $\rightarrow$ Main topic of this talk !

$q\bar{q}g$ part of the transverse photon wave-function



Light-front quantization of QCD+QED
$\Rightarrow$ instantaneous interactions in x^+.

$q\bar{q}g$ part of the longitudinal photon wave-function



In Light-front quantization : no longitudinal photon in the Hilbert space, only in instantaneous Coulomb interactions.

$\Rightarrow$ Effective $\gamma_L^* \to q\bar{q}$ vertex: part of the instantaneous $e \to eq\bar{q}$ vertex.

$\Rightarrow$ No instantaneous diagrams for the longitudinal photon case.

$q\bar{q}g$ part of the photon wave-function

$$\left|\gamma^*_{T,L}\left(q^+,Q^2,(\lambda)\right)_H\right\rangle_{q\bar{q}g} = \frac{eg}{2(2\pi)} \int_0^1 \frac{\mathrm{d}z_0}{\sqrt{z_0}} \int_0^1 \frac{\mathrm{d}z_1}{\sqrt{z_1}} \int_0^1 \frac{\mathrm{d}z_2}{z_2}\, \delta(z_0+z_1+z_2-1) \int \frac{\mathrm{d}^2\mathbf{x}_0}{(2\pi)^2} \int \frac{\mathrm{d}^2\mathbf{x}_1}{(2\pi)^2}$$

$$\times \int \frac{\mathrm{d}^2\mathbf{x}_2}{(2\pi)^2} \sum_{h_0,\lambda_2} \Phi^{q\bar{q}g}_{T,L}\left(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_0,z_1,z_2,h_0,\lambda_2,(\lambda)\right) \sum_{A_0,A_1,a} \left(T^a\right)_{A_0A_1}$$

$$\times \sum_f e_f\; b^\dagger(\mathbf{x}_0,z_0q^+,h_0,A_0,f)\, d^\dagger(\mathbf{x}_1,z_1q^+,-h_0,A_1,f)\, a^\dagger(\mathbf{x}_2,z_2q^+,\lambda_2,a)|0\rangle$$

$q\bar{q}g$ part of the longitudinal photon wave-function

$$\Phi_L^{q\bar{q}g}\left(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_0,z_1,z_2,h_0,\lambda_2\right)=2iQ\,\mathrm{K}_0(QX_3)\Bigg\{z_1(1-z_1)\left[1-\frac{z_2}{(1-z_1)}\left(\frac{1-2h_0\,\lambda_2}{2}\right)\right]\frac{\varepsilon^*_{\lambda_2}\cdot\mathbf{x}_{20}}{x_{20}^2}$$
$$-z_0(1-z_0)\left[1-\frac{z_2}{(1-z_0)}\left(\frac{1+2h_0\,\lambda_2}{2}\right)\right]\frac{\varepsilon^*_{\lambda_2}\cdot\mathbf{x}_{21}}{x_{21}^2}\Bigg\}$$

with the notation:

$$X_3^2 \quad = \quad z_1\,z_0\,x_{10}^2+z_2\,z_0\,x_{20}^2+z_2\,z_1\,x_{21}^2$$

$q\bar{q}g$ part of the transverse photon wave-function

$$\Phi_T^{q\bar{q}g}\left(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_0,z_1,z_2,h_0,\lambda_2,\lambda\right)=\frac{QX_3\,\mathrm{K}_1(QX_3)}{X_3^2}$$

$$\times\Bigg\{z_1(1-z_1)[1-2z_1+2h_0\,\lambda]\,\varepsilon_\lambda\cdot\left(\mathbf{x}_{10}-\frac{z_2}{1-z_1}\mathbf{x}_{20}\right)\left[1-\frac{z_2}{(1-z_1)}\left(\frac{1-2h_0\,\lambda_2}{2}\right)\right]\frac{\varepsilon^*_{\lambda_2}\cdot\mathbf{x}_{20}}{x_{20}^2}$$

$$-z_0(1-z_0)[1-2z_0-2h_0\,\lambda]\,\varepsilon_\lambda\cdot\left(\mathbf{x}_{01}-\frac{z_2}{1-z_0}\mathbf{x}_{21}\right)\left[1-\frac{z_2}{(1-z_0)}\left(\frac{1+2h_0\,\lambda_2}{2}\right)\right]\frac{\varepsilon^*_{\lambda_2}\cdot\mathbf{x}_{21}}{x_{21}^2}$$

$$-z_0\,z_1\,z_2\,\delta_{\lambda,\lambda_2}\left[\frac{\delta_{\lambda,-2h_0}}{1-z_1}-\frac{\delta_{\lambda,2h_0}}{1-z_0}\right]\Bigg\}$$

Virtual corrections from probability conservation

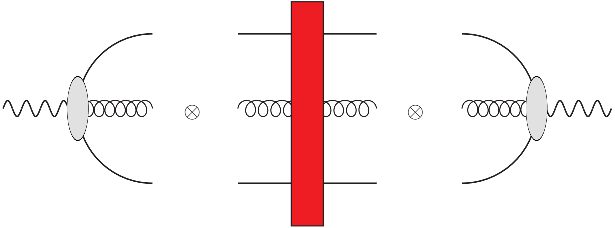
Unitarity $\Rightarrow q\bar{q} + q\bar{q}g$ components of the wave function up to order $\mathcal{O}(eg)$: Same normalization as $q\bar{q}$ component at order $\mathcal{O}(e)$.

$$\left|\Phi^{LO}_{T,L}\left(\mathbf{x}_0,\mathbf{x}_1,1-z_1,z_1,(h_0),(\lambda)\right)\right|^2 = \left|\Phi^{q\bar{q}}_{T,L}\left(\mathbf{x}_0,\mathbf{x}_1,1-z_1,z_1,h_0,(\lambda)\right)\right|^2$$
$$+\left(1-\frac{1}{N_c^2}\right)\bar{\alpha}\int_0^{1-z_1}\frac{\mathrm{d}z_2}{z_2}\int\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\sum_{\lambda_2}\left|\Phi^{q\bar{q}g}_{T,L}\left(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,1-z_1-z_2,z_1,z_2,h_0,\lambda_2,(\lambda)\right)\right|^2$$

With the notation:

$$\bar{\alpha} = \frac{N_c}{\pi}\alpha_s = \frac{N_c\, g^2}{(2\pi)^2}$$

DIS on a classical gluon shockwave field $\mathcal{A}$



$+$ virtual corrections.

$$\sigma^{\gamma}_{T,L}[\mathcal{A}] = 2\,\frac{2N_c\,\alpha_{em}}{(2\pi)^2}\sum_f e_f^2 \int \mathrm{d}^2\mathbf{x}_0 \int \mathrm{d}^2\mathbf{x}_1 \int_0^1 \mathrm{d}z_1 \Bigg\{ \Big[1-\mathcal{S}_{01}[\mathcal{A}]\Big]\, \mathcal{I}^{LO}_{T,L}(x_{01},1-z_1,z_1)$$
$$+\bar{\alpha}\int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\int_{z_f}^{1-z_1}\frac{\mathrm{d}z_2}{z_2}\Big[\mathcal{S}_{01}[\mathcal{A}]-\mathcal{S}_{02}[\mathcal{A}]\,\mathcal{S}_{21}[\mathcal{A}]\Big]\,\mathcal{I}^{NLO}_{T,L}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,1-z_1-z_2,z_1,z_2)\Bigg\}$$

z_f: IR cut-off.

Longitudinal NLO impact factor

$$\mathcal{I}_L^{NLO}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_0,z_1,z_2)=4Q^2\,\mathrm{K}_0^2(QX_3)\Bigg\{z_1^2(1-z_1)^2\frac{\mathcal{P}\left(\frac{z_2}{1-z_1}\right)}{x_{20}^2}$$
$$+z_0^2(1-z_0)^2\frac{\mathcal{P}\left(\frac{z_2}{1-z_0}\right)}{x_{21}^2}-2z_1(1-z_1)z_0(1-z_0)\left[1-\frac{z_2}{2(1-z_1)}-\frac{z_2}{2(1-z_0)}\right]\left(\frac{\mathbf{x}_{20}\cdot\mathbf{x}_{21}}{x_{20}^2\,x_{21}^2}\right)\Bigg\}$$

DGLAP quark to gluon splitting function:

$$\mathcal{P}(z)=\frac{1}{2}\left[1+(1-z)^2\right]$$

Transverse NLO impact factor

$$\mathcal{I}_T^{NLO}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_0,z_1,z_2)=\left[\frac{QX_3\,\mathrm{K}_1\big(QX_3\big)}{X_3^2}\right]^2\Bigg\{z_1^2(1-z_1)^2\left[z_1^2+(1-z_1)^2\right]\left(\mathbf{x}_{10}-\frac{z_2}{1-z_1}\mathbf{x}_{20}\right)^2\frac{\mathcal{P}\left(\frac{z_2}{1-z_1}\right)}{x_{20}^2}$$

$$+z_0^2(1-z_0)^2\left[z_0^2+(1-z_0)^2\right]\left(\mathbf{x}_{01}-\frac{z_2}{1-z_0}\mathbf{x}_{21}\right)^2\frac{\mathcal{P}\left(\frac{z_2}{1-z_0}\right)}{x_{21}^2}$$

$$+2z_1(1-z_1)z_0(1-z_0)\left[z_1(1-z_0)+z_0(1-z_1)\right]\left(\mathbf{x}_{10}-\frac{z_2}{1-z_1}\mathbf{x}_{20}\right)\cdot\left(\mathbf{x}_{01}-\frac{z_2}{1-z_0}\mathbf{x}_{21}\right)$$

$$\times\left[1-\frac{z_2}{2(1-z_1)}-\frac{z_2}{2(1-z_0)}\right]\left(\frac{\mathbf{x}_{20}\cdot\mathbf{x}_{21}}{x_{20}^2\;x_{21}^2}\right)$$

$$+\frac{z_0\,z_1\,z_2^2\,(z_0-z_1)^2}{(1-z_1)(1-z_0)}\;\frac{\left(\mathbf{x}_{20}\wedge\mathbf{x}_{21}\right)^2}{x_{20}^2\;x_{21}^2}+z_0\,z_1^2\,z_2\left[\frac{z_0\,z_1}{(1-z_1)}+\frac{(1-z_1)^2}{(1-z_0)}\right]\left(\mathbf{x}_{10}-\frac{z_2}{1-z_1}\mathbf{x}_{20}\right)\cdot\left(\frac{\mathbf{x}_{20}}{x_{20}^2}\right)$$

$$+z_0^2\,z_1\,z_2\left[\frac{z_0\,z_1}{(1-z_0)}+\frac{(1-z_0)^2}{(1-z_1)}\right]\left(\mathbf{x}_{01}-\frac{z_2}{1-z_0}\mathbf{x}_{21}\right)\left(\frac{\mathbf{x}_{21}}{x_{21}^2}\right)+\frac{z_0^2\,z_1^2\,z_2^2}{2}\left[\frac{1}{(1-z_1)^2}+\frac{1}{(1-z_0)^2}\right]\Bigg\}$$

High-energy factorization

Choice of high-energy factorization scheme:

- gluons with $k^+ > z_f\, q^+$: kept into the NLO impact factor
- gluons with $k^+ < z_f\, q^+$: put into the shockwave field $\mathcal{A}$ of the target

see *e.g* Balitsky, Chirilli (2007)

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Physical minimal cut-off set by the target:

$$z_f\, q^+ > k^+_{min} = \frac{Q_0^2}{2P^-} = x\, \frac{Q_0^2}{Q^2}\, q^+$$

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$\Rightarrow$ Range for LL evolution from the target to the factorization scale:

$$Y_f^+ = \log\left(\frac{z_f\, q^+}{k^+_{\text{min}}}\right) = \log\left(\frac{z_f\, Q^2}{x\, Q_0^2}\right)$$

$\rightarrow$ Not a rapidity range, and not $\log(1/x)$ either, beyond LL

Final result

$$\sigma^{\gamma}_{T,L} = 2\,\frac{2N_c\,\alpha_{em}}{(2\pi)^2}\sum_f e_f^2 \int \mathrm{d}^2\mathbf{x}_0 \int \mathrm{d}^2\mathbf{x}_1 \int_0^1 \mathrm{d}z_1 \Bigg\{ \mathcal{I}^{LO}_{T,L}(\mathbf{x}_0,\mathbf{x}_1,1-z_1,z_1)$$

$$\times\left[1-\langle\mathcal{S}_{01}\rangle_{Y_f^+} + \bar{\alpha}\log\left(\frac{1-z_1}{z_f}\right)\int\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\frac{x_{01}^2}{x_{02}^2\,x_{21}^2}\,\langle\mathcal{S}_{01}-\mathcal{S}_{02}\mathcal{S}_{21}\rangle_{Y_f^+}\right]$$

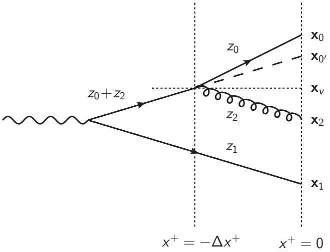
$$+\bar{\alpha}\int\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\langle\mathcal{S}_{01}-\mathcal{S}_{02}\,\mathcal{S}_{21}\rangle_{Y_f^+}\int_{z_f}^{1-z_1}\frac{\mathrm{d}z_2}{z_2}\,\Delta\mathcal{I}^{NLO}_{T,L}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_1,z_2)\Bigg\}$$

with

$$\Delta\mathcal{I}^{NLO}_{T,L}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_1,z_2) = \mathcal{I}^{NLO}_{T,L}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,1-z_1-z_2,z_1,z_2) - \mathcal{I}^{NLO}_{T,L}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,1-z_1,z_1,0)$$

Evolution in Y_f^+ (or equivalently z_f) given by B-JIMLWK evolution at LL accuracy as expected.

Transverse recoil effects



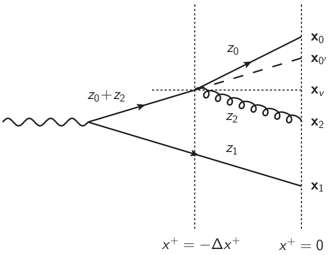
Including recoil effects, the parent dipole is not $\mathbf{x}_{01}$ but $\mathbf{x}_{0'1}$ for the diagram (a), with

$$\mathbf{x}_{0'} = \frac{z_0\,\mathbf{x}_0 + z_2\,\mathbf{x}_2}{z_0 + z_2}$$

In general: parent parton position always at the barycenter of the daughter partons, when including transverse recoil.

cf. talks by Matthias Burkardt and Markus Diehl

Formation time of multiparticle states



From diagram (a): the first expression obtained for X_3 is

$$X_3^2\Big|_{(a)} = z_1(1-z_1)\, x_{10'}^2 + \frac{z_2\, z_0}{(z_2+z_0)}\, x_{20}^2$$

$\Rightarrow$ sum of the formation times associated with the two splitting, up to a factor $2q^+$.

Formation time of multiparticle states

Little miracle: same argument in the Bessel functions for all diagrams, including instantaneous ones!

$$X_3^2\Big|_{(a)} = X_3^2\Big|_{(b)} = X_3^2\Big|_{(c)} = X_3^2\Big|_{(d)} = z_1\, z_0\, x_{10}^2 + z_2\, z_0\, x_{20}^2 + z_2\, z_1\, x_{21}^2$$

$\Rightarrow$ universal expression for the formation time of a 3−partons state.

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$\Rightarrow$ universal expression for the formation time of a $3-$partons state.

Generalization:
Formation time for a $n-$partons state from a single parton:
$\tau_{\mathrm{form},\ n\ \mathrm{part}} = 2q^+\, X_n^2$, with

$$X_n^2 = \sum_{\substack{i,j=0\\ i<j}}^{n-1} z_i\, z_j\, x_{ij}^2$$

independently of the dynamical mechanism.

Formation time of multiparticle states

Impact parameter of the parton cascade, including recoil effects ($\rightarrow$ position of the parent photon):

$$\mathbf{x}_b = \sum_{i=0}^{n-1} z_i \, \mathbf{x}_i$$

Alternative formula for the formation time variable:

$$X_n^2 = \sum_{\substack{i,j=0 \\ i<j}}^{n-1} z_i \, z_j \, x_{ij}^2 = \sum_{i=0}^{n-1} z_i \, x_{ib}^2$$

Back to the subtraction of LL BK from NLO σ_L^γ

Low z_2 contribution to σ_L^γ at NLO:

$$\sim \bar{\alpha}\frac{\mathrm{d}z_2}{z_2}\int\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\frac{x_{01}^2}{x_{02}^2\,x_{21}^2}\,\mathrm{K}_0^2(QX_3)\,\langle\mathcal{S}_{01}-\mathcal{S}_{02}\,\mathcal{S}_{21}\rangle$$

for $z_2 \ll z_1,\ 1-z_1$.

Low z_2 term used to subtract LL from σ_L^γ at NLO:

$$\sim \bar{\alpha}\frac{\mathrm{d}z_2}{z_2}\int\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\frac{x_{01}^2}{x_{02}^2\,x_{21}^2}\,\mathrm{K}_0^2\left(Q\sqrt{z_1(1-z_1)x_{01}^2}\right)\,\langle\mathcal{S}_{01}-\mathcal{S}_{02}\,\mathcal{S}_{21}\rangle$$

$\Rightarrow$ Mismatch at low z_2 in the regime
$z_1(1-z_1)x_{01}^2 \ll z_2x_{02}^2 \simeq z_2x_{12}^2$, where $X_3^2 \simeq z_2x_{02}^2 \simeq z_2x_{12}^2$.

Back to the subtraction of LL BK from NLO σ_L^γ

In the regime $z_2 \ll z_1,\ 1-z_1$ and $z_1(1-z_1)x_{01}^2 \ll z_2 x_{02}^2 \simeq z_2 x_{12}^2$:

- $\mathrm{K}_0(QX_3)$ is exponentially smaller than $\mathrm{K}_0\left(Q\sqrt{z_1(1-z_1)x_{01}^2}\right)$
- and no contribution to leading logs is present in σ_L^γ at NLO.

$\Rightarrow$ More leading logs subtracted with the BK equation than present in σ_L^γ (and σ_T^γ).

Incorrect treatment in a kinematical regime parametrically narrow, but quantitatively important:

that's where DGLAP physics sits!

Link with the problems of NLL BK and BFKL

BK and BFKL usually derived in strict Regge kinematics:

- strong ordering in k^+ (or k^-, or rapidity)
- all $\mathbf{k}$'s (or dipole sizes) of the same order

and kinematical approximations are performed accordingly.

Problem: unrestricted integration over $\mathbf{k}$ or $\mathbf{x}$ in BFKL and BK
$\Rightarrow$ Second assumption not consistent!

This is the origin of the largest NLL, NNLL and so on corrections in the BFKL and BK equations.

The whole tower of such large higher order corrections can be resummed into the LL equations by treating more carefully the kinematics.

Previous works on that resummation

That resummation of large higher order corrections is done for BFKL in momentum space by imposing a kinematical constraint in the kernel.
Ciafaloni (1988)
Kwieciński, Martin, Sutton (1996)
Andersson, Gustafson, Kharraziha, Samuelsson (1996)

That kinematical constraint is part of the full treatment of NLL BFKL, together with the resummation of other (less) large corrections (done in momentum or in Mellin space).
Salam (1998)
Ciafaloni, Colferai, Salam, Staśto (1999-2007)
Altarelli, Ball, Forte (2000-2008)

First attempt in mixed space:
Motyka, Staśto (2009)
⇒ General idea correct but wrong implementation of virtual terms.

Kinematically constrained BK equation (kcBK)

With improved kinematics for the real term, and the virtual term obtained by probability conservation:

$$\partial_{Y_f^+} \langle \mathcal{S}_{01} \rangle_{Y_f^+} = \bar{\alpha} \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2\, x_{21}^2}\, \theta(Y_f^+ - \Delta_{012}) \Big\{ \Big\langle \mathcal{S}_{02}\mathcal{S}_{21} - \frac{1}{N_c^2} \mathcal{S}_{01} \Big\rangle_{Y_f^+ - \Delta_{012}} - \Big(1 - \frac{1}{N_c^2}\Big) \langle \mathcal{S}_{01} \rangle_{Y_f^+} \Big\}$$

with the notation

$$\Delta_{012} = Max \left\{ 0,\ \log\left(\frac{x_{02}^2}{x_{01}^2}\right),\ \log\left(\frac{x_{21}^2}{x_{01}^2}\right) \right\}$$

G.B., *in preparation*

Only gluon emission at large transverse distance is modified, and regime of very large transverse distances completely removed.

This should slow down significantly the BK evolution!

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- Other DIS observables (diffractive, exclusive, ...) with dipole factorization?
- NLO result gives interesting insight into exact kinematics of parton cascades in mixed space.

Conclusions about kcBK

- Strict (naive) Regge kinematics leads to large higher corrections to the low x evolution kernels and to the impact factors of all observables.
- Kinematical improvement solves those problems: kcBK equation.
- Warning! Kinematical constraint is factorization scheme dependent. Here, only *rigid cut-off* in k^+ has been considered.
- What about kinematical constraint for *conformal dipole* factorization scheme?
- The state of the art for phenomenology should now move from rcBK to rckcBK!

However, before full NLO/NLL practical studies, one should probably also resum other (less) large higher order corrections.