Transverse (Spin) Structure of Hadrons

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Deeply virtual Compton scattering (DVCS)  
\[ \rightarrow \] Generalized parton distributions (GPDs)  
\[ \rightarrow \] 'transverse imaging'  
Chromodynamik lensing and \( \perp \) single-spin asymmetries (SSA)  

\[
\begin{align*}
\text{transverse distortion of PDFs} & \quad + \quad \text{final state interactions} \\
\{ & \quad \Rightarrow \\
\text{SSA in} & \quad \gamma N \rightarrow \pi + X \\
\text{quark-gluon correlations} & \quad \rightarrow \perp \text{force on } q \text{ in DIS} \\
\text{Summary} & 
\end{align*}
\]
3 D imaging — join the experience!
virtual Compton scattering: $\gamma^* p \rightarrow \gamma p$ (actually: $e^- p \rightarrow e^- \gamma p$)

‘deeply’: $-q_\gamma^2 \gg M_p^2, |t| \rightarrow$ Compton amplitude dominated by (coherent superposition of) Compton scattering off single quarks

$\rightarrow$ only difference between form factor (a) and DVCS amplitude (b) is replacement of photon vertex by two photon vertices connected by quark (energy denominator depends on quark momentum fraction $x$)

$\rightarrow$ DVCS amplitude provides access to momentum-decomposition of form factor = Generalized Parton Distribution (GPDs).

$$\int dx H_q(x, \xi, t) = F_1^q(t) \quad \int dx E_q(x, \xi, t) = F_2^q(t)$$
Deeply Virtual Compton Scattering (DVCS)

- Virtual Compton scattering: $\gamma^* p \rightarrow \gamma p$ (actually: $e^- p \rightarrow e^- \gamma p$)
- ‘deeply’: $-q^2_\gamma \gg M^2_p, |t| \rightarrow$ Compton amplitude dominated by (coherent superposition of) Compton scattering off single quarks
- Only difference between form factor (a) and DVCS amplitude (b) is replacement of photon vertex by two photon vertices connected by quark (energy denominator depends on quark momentum fraction $x$)
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\[ \int dx H_q(x, \xi, t) = F_1^q(t) \]
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Future experiments
JLab@12GeV, COMPASS II, EIC, PANDA/FAIR
form factors: \( \frac{FT}{\leftrightarrow} \rho(\vec{r}) \)

\( GPDs(x, \vec{\Delta}) \): form factor for quarks with momentum fraction \( x \)

suitable FT of \( GPDs \) should provide spatial distribution of quarks with momentum fraction \( x \)

careful: cannot measure longitudinal momentum \( (x) \) and longitudinal position simultaneously (Heisenberg)

consider purely transverse momentum transfer

**Impact Parameter Dependent Quark Distributions**

\[
q(x, \mathbf{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} H(x, \xi = 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}
\]

\( q(x, \mathbf{b}_\perp) \) = parton distribution as a function of the separation \( \mathbf{b}_\perp \) from the transverse center of momentum \( \mathbf{R}_\perp \equiv \sum_{i \in q, g} r_{\perp, i} x_i \)


- No relativistic corrections (Galilean subgroup!)

- corollary: interpretation of 2d-FT of \( F_1(Q^2) \) as charge density in transverse plane also free of relativistic corrections

- probabilistic interpretation
Impact parameter dependent quark distributions

$q(x, b_\perp)$ for unpol. p

$q(x, b_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} H(x, 0, -\Delta_\perp^2) e^{-i b_\perp \cdot \Delta_\perp}$

- $x =$ momentum fraction of the quark
- $\vec{b} = \perp$ distance of quark from $\perp$ center of momentum
- small $x$: large ’meson cloud’
- larger $x$: compact ’valence core’
- $x \to 1$: active quark becomes center of momentum
  $\rightarrow \vec{b}_\perp \to 0$ (narrow distribution) for $x \to 1$
Impact parameter dependent quark distributions

proton polarized in \( +\hat{x} \) direction

no axial symmetry!

\[
q(x, b_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} H_q(x, 0, -\Delta^2_\perp) e^{-i b_\perp \cdot \Delta_\perp} \\
- \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} E_q(x, 0, -\Delta^2_\perp) e^{-i b_\perp \cdot \Delta_\perp}
\]

Physics: relevant density in leading twist

DIS is \( j^+ \equiv j^0 + j^3 \) and left-right asymmetry from \( j^3 \)

Impact parameter dependent quark distributions

proton polarized in $+\hat{x}$ direction

\[ q(x, b_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} H_q(x, 0, -\Delta_\perp^2) e^{-ib_\perp \cdot \Delta_\perp} \]
\[ -\frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} E_q(x, 0, -\Delta_\perp^2) e^{-ib_\perp \cdot \Delta_\perp} \]

sign & magnitude of the average shift
model-independently related to p/n anomalous magnetic moments:

\[ \langle b_y^q \rangle \equiv \int dx \int d^2 b_\perp q(x, b_\perp) b_y \]
\[ = \frac{1}{2M} \int dx E_q(x, 0, 0) = \frac{\kappa_q}{2M} \]
Impact parameter dependent quark distributions

\[ \langle b^q \rangle \equiv \int dx \int d^2 b_{\perp} q(x, b_{\perp}) b_y = \frac{1}{2M} \int dx E_q(x, 0, 0) = \frac{\kappa_q}{2M} \]

\( \kappa^p = 1.913 = \frac{2}{3} \kappa_u^p - \frac{1}{3} \kappa_d^p + \ldots \)
- \( u \)-quarks: \( \kappa_u^p = 2\kappa_p + \kappa_n = 1.673 \)
- shift in \( +\hat{y} \) direction
- \( d \)-quarks: \( \kappa_d^p = 2\kappa_n + \kappa_p = -2.033 \)
- shift in \( -\hat{y} \) direction
- \( \langle b_y^q \rangle = \mathcal{O}(\pm 0.2 \text{fm}) \)
Impact parameter dependent quark distributions

- Sign & magnitude of the average shift
- Model-independently related to p/n anomalous magnetic moments:

\[ \langle b_y^q \rangle \equiv \int dx \int d^2b_q(x, b_{\perp})b_y \]

\[ = \frac{1}{2M} \int dx E_q(x, 0, 0) = \frac{\kappa_q}{2M} \]

- Lattice QCD (QCDSF): lowest moment
Angular Momentum Carried by Quarks

transverse images ↔ Ji relation for quark angular momentum:

- \( J^x_q = m_N \int dx \, x r^y q(x, r_\perp) \) with \( b^y = r^y - \frac{1}{2m_N} \), where \( q(x, r_\perp) \) is distribution relative to CoM of whole nucleon
- recall: \( q(x, b_\perp) \) for nucleon polarized in \(+\hat{\mathbf{x}}\) direction

\[
q(x, b_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} H_q(x, 0, -\Delta^2_\perp) e^{-i b_\perp \cdot \Delta_\perp} - \frac{1}{2M_N} \frac{\partial}{\partial b_y} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} E_q(x, 0, -\Delta^2_\perp) e^{-i b_\perp \cdot \Delta_\perp}
\]

\[
\Rightarrow J^x_q = M_N \int dx \, x r^y q(x, r_\perp) = \int dx \, x \left( m_N b^y + \frac{1}{2} \right) q(x, r_\perp) = \frac{1}{2} \int dx \, x [H(x, 0, 0) + E(x, 0, 0)]
\]

- X.Ji(1996): rotational invariance ⇒ apply to all components of \( \vec{J}_q \)
- partonic interpretation exists only for \( \perp \) components!
Angular Momentum Carried by Quarks

lattice: QCDSF

\[ J^q = \frac{1}{2} \int dx \, x [H(x, 0, 0) + E(x, 0, 0)] \]

\[ L^q = J^q - \frac{1}{2} \Delta \Sigma^q \]

- no disconnected quark loops
- chiral extrapolation
Transverse Imaging in Momentum Space

TMDs

- **Transverse Momentum Dependent Parton Distributions**
- 8 structures possible at leading twist (only 3 for PDFs)
- $f_{1T}^\perp$ and $h_1^\perp$ require both **orbital angular momentum** and **final state interaction**
- can be measured in semi-inclusive deep-inelastic scattering (SIDIS) & Drell-Yan (DY) $q\bar{q} \rightarrow \mu^+\mu^-$

experiments

JLab@6GeV & 12GeV, HERMES, COMPASS I & II, RHIC, EIC, FAIR/PANDA
GPD $\leftrightarrow$ Single Spin Asymmetries (SSA)

Sivers $f_{1T}^{\perp}$ in semi-inclusive deep-inelastic scattering (SIDIS) $\gamma p \rightarrow \pi X$

- $u,d$ distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign 'determined' by $\kappa_u$ & $\kappa_d$
- attractive FSI deflects active quark towards the CoM
- FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction → 'chromodynamic lensing'

$\Rightarrow \quad \kappa_p, \kappa_n \leftrightarrow$ sign of SSA!!!!!!! MB, PRD 69, 074032 (2004)

- confirmed by HERMES (and recent COMPASS) $p$ data; consistent with vanishing isoscalar Sivers (COMPASS) → G. Schnell
compare FSI for ’red’ $q$ that is being knocked out of nucleon with ISI for ’anti-red’ $\bar{q}$ that is about to annihilate with a ’red’ target $q$

FSI in SIDIS

- knocked-out $q$ ’red’
- $\rightarrow$ spectators ’anti-red’
- $\rightarrow$ interaction between knocked-out quark and spectators attractive

ISI in DY

- incoming $\bar{q}$ ’anti-red’
- $\rightarrow$ struck target $q$ ’red’
- $\rightarrow$ spectators also ’anti-red’
- $\rightarrow$ interaction between incoming $\bar{q}$ and spectators repulsive

\[ f_{1T}^\perp(x, \mathbf{k}_\perp)_{DY} = -f_{1T}^\perp(x, \mathbf{k}_\perp)_{SIDIS} \quad \text{and} \quad h_1^\perp(x, \mathbf{k}_\perp)_{DY} = -h_1^\perp(x, \mathbf{k}_\perp)_{SIDIS} \]

critical test of TMD factorization approach
Sign of Boer-Mulders Function

- Transversity distribution in unpolarized target described by chirally odd GPD $\bar{E}_T$ (M. Diehl & P. Haegler ’05)
- $\bar{E}_T > 0$ for $u$ & $d$ (QCDSF)
- Connection $h^\perp_1(x, k_\perp) \leftrightarrow \bar{E}_T$ similar to $f^\perp_{1T}(x, k_\perp) \leftrightarrow E$ (MB ’05)
- $h^\perp_1(x, k_\perp) < 0$ for $u/p$, $d/p$, $u/\pi$, $\bar{d}/\pi$
- $h^\perp_1_{SIDIS} = -h^\perp_1_{DY}$

Experiments (no polarization needed!): HERMES, COMPASS, RHIC, JLab@12GeV, FAIR/PANDA, EIC
Primordial Quark Transversity Distribution

→ \perp \text{ quark pol.}
Probing BM Function in Tagged SIDIS

Flip of Quark Transverse Spin Component

when $\perp$ pol. quark absorbs $\gamma^*$, $\perp$ polarization
- gets reduced in size
- tilted symmetrically w.r.t. normal of lepton scattering plane
Primordial Quark Transversity Distribution

→ ⊥ quark pol.
Quark Transversity Distribution after $\gamma^*$ Absorption

→ $\perp$ quark pol.

lepton scattering plane
$\perp$ momentum (of $q$) due to FSI

$\perp$ quark pol.

$k^q_\perp$ due to FSI

lepton scattering plane
additional $\perp$ momentum (of $\pi$) due to Collins effect

Collins: favored $\pi$ momentum preferentially to left (quark spin up)
net $k^\pi_\perp$ (FSI + Collins)

- $k_\perp$ due to Collins
- $k^q_\perp$ due to FSI
- net $k^q_\perp$

lepton scattering plane
net $k^\pi_\perp$ (FSI + Collins)

$\cos 2\pi$ modulation of $k^\pi_\perp$
higher twist in polarized DIS

- $\sigma_{LL} \propto g_1 - \frac{2M}{\nu} g_2$
- $g_1 = \frac{1}{2} \sum_q e_q^2 g_1^q$ with $g_1^q = q^+(x) + \bar{q}^+(x) - q^-(x) - \bar{q}^-(x)$
- $g_2$ involves quark-gluon correlations

→ no parton interpret. as difference between number densities for $g_2$

- for $\perp$ pol. target, $g_1 \& g_2$ contribute equally

$$\sigma_{LT} \propto g_T \equiv g_1 + g_2$$

→ 'clean' separation between $g_2$ and $\frac{1}{Q^2}$ corrections to $g_1$

What can we learn from $g_2$?

- $g_2 = g_2^{WW} + \bar{g}_2$ with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3 \int dx x^2 \bar{g}_2(x) = \frac{1}{2MP^2 S_x} \left\langle P, S | \bar{q}(0) g G^+(0) \gamma^+ q(0) | P, S \right\rangle$$
\[
\begin{align*}
\bar{d}_2 &\equiv 3 \int dx \ x^2 \tilde{g}_2(x) = \frac{1}{2M^2P + 2} \left\langle P, S \mid \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right\rangle_{P, S} \\
\sqrt{2}G^{+y} &= G^{0y} + G^{zy} = -E^y + B^x
\end{align*}
\]
\[ d_2 \equiv 3 \int dx \, x^2 \bar{g}_2(x) = \frac{1}{2MP^2 + S_x} \langle P, S \mid \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \mid P, S \rangle \]

**color Lorentz force**

\[
\sqrt{2} G^{+y} = G^{0y} + G^{zy} = -E^y + B^x = -\left( \vec{E} + \vec{v} \times \vec{B} \right)^y \quad \text{for} \quad \vec{v} = (0, 0, -1)
\]
\[
d_2 \equiv 3 \int dx \, x^2 \bar{g}_2(x) = \frac{1}{2MP^2 + 2Sx} \langle P, S \mid \bar{q}(0)gG^{+y}(0)\gamma^+q(0) \mid P, S \rangle
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\]

\(\rightarrow d_2 \leftrightarrow \) average **color Lorentz force** acting on quark moving with \(v = c\) in \(-\hat{z}\) direction in the instant after being struck by \(\gamma^*\)

\[
\langle F^y \rangle = -2M^2d_2 = -\frac{M}{P^2 + 2Sx} \langle P, S \mid \bar{q}(0)gG^{+y}(0)\gamma^+q(0) \mid P, S \rangle
\]

cf. Qiu-Sterman matrix element \(\langle k^y \rangle \equiv \int_0^1 dx \int d^2k_\perp \, k^2_\perp f_{1T}(x, k^2_\perp)\)

\[
\langle k^y \rangle = -\frac{1}{2p^+} \left\langle P, S \bigg| \bar{q}(0) \int_0^\infty dx^- \, gG^{+y}(x^-)\gamma^+q(0) \bigg| P, S \right\rangle
\]

semi-classical interpretation: average \(k_\perp\) in SIDIS obtained by correlating the quark density with the transverse impulse acquired from (color) Lorentz force acting on struck quark along its trajectory to (light-cone) infinity

**matrix element defining \(d_2\)** \(\leftrightarrow\) **1\(^{st}\) integration point in QS-integral**
Quark-Gluon Correlations: Interpretation

\[ d_2 \equiv 3 \int dx \ x^2 \bar{g}_2(x) = \frac{1}{2MP^2 + 2S} \left\langle P, S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P, S \right\rangle \]

**color Lorentz force**

\[
\sqrt{2} G^{+y} = G^{0y} + G^{zy} = -E^y + B^x = -\left( \vec{E} + \vec{v} \times \vec{B} \right)^y \text{ for } \vec{v} = (0, 0, -1)
\]

\[ \rightarrow d_2 \leftrightarrow \text{average color Lorentz force} \text{ acting on quark moving with } v = c \text{ in } -\hat{z} \text{ direction in the instant after being struck by } \gamma^* \]

\[ \langle F^y \rangle = -2M^2 d_2 = -\frac{M}{P^2 + 2S} \left\langle P, S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P, S \right\rangle \]

**sign of \( d_2 \leftrightarrow \perp \) imaging**

- \( \kappa_q/p \rightarrow \) sign of deformation
- \( \rightarrow \) direction of average force
- \( d^u_2 > 0, \ d^d_2 < 0 \)
- \( \text{cf. } f_{1T}^\perp u < 0, \ f_{1T}^\perp u < 0 \)

**lattice (Göckeler et al., 2005)**

\[ d^u_2 \approx 0.010, \ d^d_2 \approx -0.0056 \]

**magnitude of \( d_2 \)**

- \[ \langle F^y \rangle = -2M^2 d_2 = -10 \frac{GeV}{fm} d_2 \]
- \( \text{expect partial cancellation of forces in SSA} \)
- \[ |\langle F^y \rangle| \ll \sigma \approx 1 \frac{GeV}{fm} \]
- \( \rightarrow d_2 = O(0.01) \)
Deeply Virtual Compton Scattering $\rightarrow$ GPDs

impact parameter dependent PDFs $q(x, b_\perp)$

$E^q(x, 0, -\Delta^2_\perp) \leftrightarrow \kappa_{q/p}$ (contribution from quark flavor $q$ to anomalous magnetic moment)

$E^q(x, 0, -\Delta^2_\perp) \rightarrow \perp$ deformation of PDFs for $\perp$ polarized target

$\perp$ deformation $\leftrightarrow \text{(sign of) SSA (Sivers; Boer-Mulders)}$

parton interpretation for Ji-relation

higher-twist ($\int dx \, x^2 \bar{g}_2(x), \int dx \, x^2 \bar{e}(x)$) $\leftrightarrow \perp$ force in DIS

$\perp$ deformation $\leftrightarrow \text{(sign of) quark-gluon correlations ($\int dx \, x^2 \bar{g}_2(x), \int dx \, x^2 \bar{e}(x)$)}$

combine complementary information from deeply-virtual Compton scattering, semi-inclusive DIS & Drell-Yan to study orbital angular momentum and map the 3-d structure of hadrons
Deeply Virtual Compton Scattering (DVCS)

$Q^2$ scaling for Compton form factor (JLab)