Initial shape and final flow fluctuations in event-by-event hydrodynamics for RHIC and LHC∗

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Probing the landscape of QCD matter: The future is now

Probes:
- Collective flow
- Jet modification and quenching
- Thermal electro-magnetic radiation
- Critical fluctuations
- . . .
Small narrow bump near $\eta = 0$ due to Jacobian \[ \frac{dN}{d\eta} = \int d^2p_T \frac{p_T \cosh \eta}{\sqrt{m^2 + p_T^2 \cosh^2 \eta}} \frac{dN}{dy d^2p_T} \] (dip in $dN/d\eta$ corresponds to bump in $v_2(\eta)$)
Panta rhei: “soft ridge” = “Mach cone” = flow!

ATLAS (J. Jia), Quark Matter 2011

ALICE (J. Grosse-Oetringhaus), QM11

M. Luzum, PLB 696 (2011) 499: All long-range rapidity correlations seen at RHIC are consistent with being entirely generated by hydrodynamic flow.

- anisotropic flow coefficients $v_n$ and flow angles $\psi_n$ correlated over large rapidity range!

- in the 1% most central collisions $v_3 > v_2$
  \[ \Rightarrow \] prominent “Mach cone”-like structure!
  \[ \Rightarrow \] event-by-event eccentricity fluctuations dominate!
Event-by-event shape and flow fluctuations rule!

(Alver and Roland, PRC81 (2010) 054905)

- Each event has a different initial shape and density distribution, characterized by different set of harmonic eccentricity coefficients $\varepsilon_n$.
- Each event develops its individual hydrodynamic flow, characterized by a set of harmonic flow coefficients $v_n$ and flow angles $\psi_n$.
- At small impact parameters fluctuations ("hot spots") dominate over geometric overlap effects.

(Alver & Roland, PRC81 (2010) 054905; Qin, Petersen, Bass, Müller, PRC82 (2010) 064903)
Event-by-event shape and flow fluctuations rule!

- in the 1% most central collisions $v_3 > v_2 \implies$ prominent “Mach cone”-like structure!

- triangular flow angle uncorrelated with reaction plane and elliptic flow angles
  $\implies$ due to event-by-event eccentricity fluctuations which dominate the anisotropic flows in the most central collisions
Fluctuation-driven anisotropic flow is indeed collective!

ALICE (J. Grosse-Oetringhaus) Quark Matter 2011

Two-particle Fourier coefficients factorize \( \langle v_n \Delta(p_{T1}, p_{T2}) = v_n(p_{T1}) v_n(p_{T2}) \rangle \) as required

Factorization shown to work for \( n = 2, 3, 4, 5 \) as long as both \( p_{T1}, p_{T2} < 3 \text{ GeV/c} \) (bulk matter)
Initial-state shape fluctuations
Smearing effects from nucleon growth at high energies

Between $\sqrt{s} = 23.5$ and $7,000$ GeV, nucleon area grows by factor $O(2)$ → significant smoothing of event-by-event density fluctuations from RHIC to LHC.
Eccentricity definitions:

Define event average \{\ldots\}, ensemble average \langle\ldots\rangle

Two choices for weight function in event average:

(i) Energy density $e(x_\perp;b)$

(ii) Entropy density $s(x_\perp;b)$

Define $\sigma^2_x = \{x^2\} - \{x\}^2$, $\sigma_{xy} = \{xy\} - \{x\}\{y\}$, etc.,

where $x$, $y$ are reaction-plane coordinates ($e_x \parallel b$)

1. Standard eccentricity: $\varepsilon_s \equiv \bar{\varepsilon}_{RP} = \frac{\langle \sigma^2_y - \sigma^2_x \rangle}{\langle \sigma^2_y + \sigma^2_x \rangle}$ (calculated from RP-averaged $\langle e \rangle$ or $\langle s \rangle$)

2. Average reaction-plane eccentricity: $\langle \varepsilon_{RP} \rangle = \langle \frac{\sigma^2_y - \sigma^2_x}{\sigma^2_y + \sigma^2_x} \rangle$

3. Eccentricity of the participant-plane averaged source: $\bar{\varepsilon}_{part} = \sqrt{\langle \frac{(\sigma^2_y - \sigma^2_x)^2 + 4\sigma^2_{xy}}{\sigma^2_y + \sigma^2_x} \rangle}$

4. Average participant-plane eccentricity: $\langle \varepsilon_{part} \rangle = \sqrt{\langle \frac{(\sigma^2_y - \sigma^2_x)^2 + 4\sigma^2_{xy}}{\sigma^2_y + \sigma^2_x} \rangle}$

5. r.m.s. part.-plane eccentricity: $\varepsilon_{part}\{2\} \equiv \sqrt{\langle \varepsilon^2_{part} \rangle} = \sqrt{\langle \varepsilon_{part} \rangle^2 + \sigma^2_\varepsilon/2}$ for Gauss. fl.

6. 4th cumulant eccentricity: $\varepsilon_{part}\{4\} \equiv \left[ \langle \varepsilon^2_{part} \rangle^2 - \langle \varepsilon^4_{part} \rangle - \langle \varepsilon^2_{part} \rangle^2 \right]^{1/4} = \sqrt{\langle \varepsilon_{part} \rangle^2 - \sigma^2_\varepsilon/2}$ for Gauss. fl.
MC-Glauber eccentricities (\(e\)-weighted):

\[
\langle \epsilon_{\text{part}} \rangle
\]
\[
\epsilon_{\text{part}}\{2\}
\]
\[
\epsilon_{\text{part}}\{4\}
\]
\[
\langle \epsilon_{\text{RP}} \rangle
\]
\[
\bar{\epsilon}_{\text{part}}
\]
\[
\bar{\epsilon}_{\text{RP}}
\]

Impact parameter \(b \) (fm)
MC-KLN eccentricities ($e$-weighted):

![Graph showing eccentricities as a function of impact parameter $b$ (fm). The graph includes several curves representing different eccentricity measurements, such as $\langle \epsilon_{\text{part}} \rangle$, $\epsilon_{\text{part}}^2$, $\epsilon_{\text{part}}^4$, $\langle \epsilon_{R \rho} \rangle$, $\bar{\epsilon}_{\text{part}}$, and $\bar{\epsilon}_{R \rho}$. The x-axis represents the impact parameter in femtometers (fm), while the y-axis represents the eccentricity $\epsilon$.]

[Ulrich Heinz, CPTEIC, 31 Jan. 2012]
Initial eccentricities $\varepsilon_n(n=2-5)$ vs. impact parameter

Zhi Qiu, UH, PRC84 (2011) 024911

- Contours: $e(r, \phi) = e_0 \exp \left[ -\frac{r^2}{2\rho^2} \left( 1 + \varepsilon_n \cos(n(\phi - \psi_n)) \right) \right]$
  where $\varepsilon_n e^{in\psi_n} = -\frac{\int r dr d\phi \, r^2 e^{in\phi} e(r,\phi)}{\int r dr d\phi \, r^2 e(r,\phi)}$

- MC-KLN has larger $\varepsilon_2$ and $\varepsilon_4$, but similar $\varepsilon_5$ and almost identical $\varepsilon_3$ as MC-Glauber ($\varepsilon_{3,5}$ are purely fluctuation-driven!)
Flow fluctuations in event-by-event hydro
Flow angle distributions (0–60% centrality):

\[
\varepsilon_n e^{i n \Psi_n^{PP}} = -\int r dr d\phi r^2 e^{i n \phi} e(r,\phi) \int r dr d\phi r^2 e(r,\phi)
\]

\[
v_n e^{i n \Psi_n^{EP}} = \frac{\int d\phi_p e^{i n \phi_p} (dN/d\phi_p)}{\int d\phi_p (dN/d\phi_p)}
\]

- Angles of $\varepsilon_{3,5}$ and $\nu_{3,5}$ uncorrelated with reaction plane (Qin et al., PRC 82 (2010) 064903)
- $\nu_4$-angle $\Psi_4^{EP}$ lies (on average) in the reaction plane even though $\varepsilon_4$-angle $\Psi_4^{PP}$ points at $\pm \frac{\pi}{4} = \pm 45^\circ \implies \nu_4$ driven mostly by elliptic deformation $\varepsilon_2$, not $\varepsilon_4$. 

Correlation between flow and eccentricity angles:

\[ \psi_{2EP}^2 - \psi_{2PP}^2 \mod \frac{\pi}{2} \]

Strong angular correlation between elliptic flow and eccentricity, except near \( b=0 \).
Correlation between flow and eccentricity angles:

$$\psi_3^{EP} - \psi_3^{PP} \mod \frac{\pi}{3}$$

Four times weaker angular correlation between triangular flow and triangularity than for 2nd harmonic.
Correlation between flow and eccentricity angles:

\[ \psi_{EP}^4 - \psi_{PP}^4 \mod \frac{\pi}{4} \]

- Near-central collisions: \( \psi_{EP}^4 \) (weakly) correlated with \( \psi_{PP}^4 \) \( \iff \) \( v_4 \) driven by \( \varepsilon_4 \)
- Peripheral collisions: \( \psi_{EP}^4 \) (weakly) anti-correlated with \( \psi_{PP}^4 \) \( \iff \) \( v_4 \) driven by \( \varepsilon_2 \)
- Mid-central to mid-peripheral: no correlation between \( \psi_{EP}^4 \) and \( \psi_{PP}^4 \)
Correlation between flow and eccentricity angles:

\[ \psi_{EP}^5 - \psi_{PP}^5 \mod \frac{\pi}{5} \]

- Near-central collisions: \( \psi_{EP}^5 \) (weakly) correlated with \( \psi_{PP}^5 \) \( \Rightarrow v_5 \) driven by \( \varepsilon_5 \)
- Peripheral collisions: \( \psi_{EP}^5 \) (weakly) anti-correlated with \( \psi_{PP}^5 \) \( \Rightarrow v_5 \) strongly influenced by \( \varepsilon_{n\neq5} \)
- Mid-central to mid-peripheral: no correlation between \( \psi_{EP}^5 \) and \( \psi_{PP}^5 \)
Higher harmonic flows and associated eccentricities: $v_2$ vs. $\varepsilon_2$ (MC-KLN, $e$-weighted)

- Slightly non-linear dependence of $v_2$ on $\varepsilon_2$, especially in central and very peripheral collisions
- In the most peripheral centrality classes, slope of $v_2(\varepsilon_2)$ decreases
Higher harmonic flows and associated eccentricities: $v_3$ vs. $\varepsilon_3$ (MC-KLN, $e$-weighted)

- Slope of $v_3(\varepsilon_3)$ and value of $v_3/\varepsilon_3$ depend on centrality class.

- Non-zero triangular flow $v_3 \sim 1\% - 2\%$ even for zero triangularity $\varepsilon_3$.

  $\Rightarrow$ other (odd) harmonic eccentricity coefficients feed into $v_3$. 
Higher harmonic flows and associated eccentricities: $v_4$ vs. $\varepsilon_4$ (MC-KLN, $e$-weighted)

- Correlation between $v_4$ and $\varepsilon_4$ strongly centrality dependent
- In mid-central and peripheral collisions, $v_4$ is mostly generated by $\varepsilon_{n\neq4}$, in particular $\varepsilon_2$
- Even in central collisions, other $\varepsilon_{n\neq4}$ feed into $v_4$, generating non-zero $v_4$ for zero $\varepsilon_4$
Higher harmonic flows and associated eccentricities: $v_5$ vs. $\varepsilon_5$ (MC-KLN, $e$-weighted)

- Correlation between $v_5$ and $\varepsilon_5$ strongly centrality dependent
- In mid-central and peripheral collisions, $v_5$ is mostly generated by $\varepsilon_{n\neq 5}$
- Even in central collisions, other $\varepsilon_{n\neq 5}$ feed into $v_5$, generating non-zero $v_5$ for zero $\varepsilon_5$
Event-by-event vs. single-shot hydro
For most centralities, eccentricity-scaled $v_{2,3}$ from e-by-e and single-shot hydro agree within 5-10%.

Agreement between $\langle v_{2,3} \rangle / \langle \varepsilon_{2,3} \rangle$ and $v_{2,3} \{2\} / \varepsilon_{2,3} \{2\}$ is excellent at all centralities.

Agreement between $v_{2} \{2\} / \varepsilon_{2} \{2\}$ and $v_{2} \{4\} / \varepsilon_{2} \{4\}$ is good over most of the centrality range, but the analog relation for triangular flow does not work (note, however, limited statistics).

Can use single-shot hydro to compute $\langle v_{2,3} \rangle / \langle \varepsilon_{2,3} \rangle = v_{2,3} \{2\} / \varepsilon_{2,3} \{2\}$.
\((\eta/s)_{\text{QGP}}\)
How to use elliptic flow for measuring $(\eta/s)_{\text{QGP}}$

Hydrodynamics converts spatial deformation of initial state $\implies$ momentum anisotropy of final state, through anisotropic pressure gradients

Shear viscosity degrades conversion efficiency

$$\varepsilon_x = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle} \implies \varepsilon_p = \frac{\langle T_{xx} - T_{yy} \rangle}{\langle T_{xx} + T_{yy} \rangle}$$

of the fluid; the suppression of $\varepsilon_p$ is monotonically related to $\eta/s$.

The observable that is most directly related to the total hydrodynamic momentum anisotropy $\varepsilon_p$ is the total ($p_T$-integrated) charged hadron elliptic flow $v_2^{\text{ch}}$:

$$\varepsilon_p = \frac{\langle T_{xx} - T_{yy} \rangle}{\langle T_{xx} + T_{yy} \rangle} \iff \sum_i \int p_T dp_T \int d\phi_p \frac{p_T^2}{d\phi_T dp_T} \cos(2\phi_p) \frac{dN_i}{d\phi_T dp_T d\phi_p} \iff v_2^{\text{ch}}$$

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How to use elliptic flow for measuring \((\eta/s)_{\text{QGP}}\) (contd.)

- **If** \(\varepsilon_p\) **saturates** before hadronization (e.g. in PbPb@LHC (?) )
  \[ \Rightarrow \ v_{2}^{\text{ch}} \approx \text{not affected by details of hadronic rescattering below} \ T_c \]

  **but:** \(v_2^{(i)}(p_T), \quad \frac{dN_i}{dyd^2p_T}\) change during hadronic phase (addl. radial flow!), and these changes depend on details of the hadronic dynamics (chemical composition etc.)

  \[ \Rightarrow \ v_2(p_T) \text{ of a single particle species not a good starting point for extracting } \eta/s \]

- **If** \(\varepsilon_p\) **does not saturate** before hadronization (e.g. AuAu@RHIC), dissipative hadronic dynamics affects not only the distribution of \(\varepsilon_p\) over hadronic species and in \(p_T\), but even the final value of \(\varepsilon_p\) itself (from which we want to get \(\eta/s\))

  \[ \Rightarrow \ \text{need hybrid code that couples viscous hydrodynamic evolution of QGP to realistic microscopic dynamics of late-stage hadron gas phase} \]

  \[ \Rightarrow \ \text{VISHNU ("Viscous Israel-Steward Hydrodynamics 'n' UrQMD")} \]

  (Song, Bass, UH, PRC83 (2011) 024912) \ Note: this paper shows that UrQMD \(\neq\) viscous hydro!
Extraction of \((\eta/s)_{\text{QGP}}\) from AuAu@RHIC


\[ 1 < 4\pi(\eta/s)_{\text{QGP}} < 2.5 \]

- All shown theoretical curves correspond to parameter sets that correctly describe centrality dependence of charged hadron production as well as \(p_T\)-spectra of charged hadrons, pions and protons at all centralities.
- \(v_2^{ch}/\varepsilon_x\) vs. \((1/S)(dN_{ch}/dy)\) is “universal”, i.e. depends only on \(\eta/s\) but (in good approximation) not on initial-state model (Glauber vs. KLN, optical vs. MC, RP vs. PP average, etc.).
- Dominant source of uncertainty: \(\varepsilon_{x}^{\text{G1}}\) vs. \(\varepsilon_{x}^{\text{KLN}}\)
- Smaller effects: early flow \(\rightarrow\) increases \(v_2^{ch}/\varepsilon\) by ~few % \(\rightarrow\) larger \(\eta/s\)

\(\text{bulk viscosity} \rightarrow\) affects \(v_2^{ch}(p_T)\), but \(\approx\) not \(v_2^{ch}\)

Zhi Qiu, UH, PRC84 (2011) 024911
Global description of AuAu@RHIC spectra and $v_2$


- $(\eta/s)_{QGP} = 0.08$ for MC-Glauber and $(\eta/s)_{QGP} = 0.16$ for MC-KLN work well for charged hadron, pion and proton spectra and $v_2(p_T)$ at all collision centralities

- Note: $T_{chem} = 165\,\text{MeV}$ reproduces the proton spectra from STAR, but not from PHENIX! Slightly incorrect chemical composition in hadronic phase? Not enough $p\bar{p}$ annihilation in UrQMD?
Global description of AuAu@RHIC spectra and $\nu_2$


- $(\eta/s)_{QGP} = 0.08$ for MC-Glauber and $(\eta/s)_{QGP} = 0.16$ for MC-KLN work well for charged hadron, pion and proton spectra and $\nu_2(p_T)$ at all collision centralities
- A purely hydrodynamic model (without UrQMD afterburner) with the same values of $\eta/s$ does almost as well (except for centrality dependence of proton $\nu_2(p_T)$) (C. Shen et al., PRC84 (2011) 044903)
  Main difference: VISHNU develops more radial flow in the hadronic phase (larger shear viscosity), pure viscous hydro must start earlier than VISHNU ($\tau_0 = 0.6$ instead of 1.05 fm/$c$), otherwise proton spectra are too steep
- These $\eta/s$ values agree with Luzum & Romatschke, PRC78 (2008), even though they used EOS with incorrect hadronic chemical composition shows robustness of extracting $\eta/s$ from total charged hadron $\nu_2$
After normalization in 0-5% centrality collisions, MC-KLN + VISHNU (w/o running coupling, but including viscous entropy production!) reproduces centrality dependence of $dN_{ch}/d\eta$ well in both AuAu@RHIC and PbPb@LHC.

- $(\eta/s)_{QGP} = 0.16$ for MC-KLN works well for charged hadron $v_2(p_T)$ and integrated $v_2$ in AuAu@RHIC, but overpredicts both by about 10-15% in PbPb@LHC.

- Similar results from predictions based on pure viscous hydro (C. Shen et al., PRC84 (2011) 044903)

- **but**: At LHC significant sensitivity of $v_2$ to initialization of viscous pressure tensor $\pi^{\mu\nu}$ (Navier-Stokes or zero) $\Rightarrow$ need pre-equilibrium model.

$\Rightarrow$ QGP at LHC not much more viscous than at RHIC!
Why is $v_2^{ch}(p_T)$ the same at RHIC and LHC?

**Answer:** Pure accident! (Kestin & Heinz EPJC61 (2009) 545)

$v_2^{\pi}(p_T)$ increases a bit from RHIC to LHC, for heavier hadrons $v_2(p_T)$ at fixed $p_T$ decreases

(radial flow pushes momentum anisotropy of heavy hadrons to larger $p_T$)

This is a hard prediction of hydrodynamics! (See also Nagle, Bearden, Zajc, NJP13 (2011) 075004)
Confirmation of increased mass splitting at LHC

Data: ALICE @ LHC, Quark Matter 2011 (symbols), PHENIX @ RHIC (shaded)

Lines: Shen et al., PRC84 (2011) 044903 (VISH2+1 + MC-KLN, $\eta/s=0.2$)

- Qualitative features of data agree with VISH2+1 predictions
- VISH2+1 does not push proton $v_2$ strongly enough to higher $p_T$, both at RHIC and LHC
- At RHIC we know that this is fixed when using VISHNU – is the same true at LHC?
Successful prediction of $v_2(p_T)$ for identified hadrons in PbPb@LHC

Data: ALICE, Quark Matter 2011
Prediction: Shen et al., PRC84 (2011) 044903 (VISH2+1)

Perfect fit in semi-peripheral collisions!

The problem with insufficient proton radial flow exists only in more central collisions

Adding the hadronic cascade (VISHNU) helps:
$v_2(p_T)$ in PbPb@LHC: ALICE vs. VISHNU

Data: ALICE, preliminary (Snellings, Krzewicki, Quark Matter 2011)
Dashed lines: Shen et al., PRC84 (2011) 044903 (VISH2+1, MC-KLN, $(\eta/s)_{QGP}=0.2$)
Solid lines: Song, Shen, UH 2011 (VISHNU, MC-KLN, $(\eta/s)_{QGP}=0.16$)

VISHNU yields correct magnitude and centrality dependence of $v_2(p_T)$ for pions, kaons and protons!

Same $(\eta/s)_{QGP} = 0.16$ (for MC-KLN) at RHIC and LHC!
Good description also of identified hadron spectra for centralities $< 50\%$

VISHNU better than VISH2+1 in central collisions (more radial flow)

Both models give too much radial flow in peripheral collisions $\implies$ initial conditions?

Both models overpredict proton yield by 50-70%!
The new “proton anomaly”: disagreement with the thermal model

Data: ALICE, preliminary (A. Kalweit, Strange Quark Matter 2011)


- “Standard” $T_{\text{chem}} = 164$ MeV reproduces strange hadrons but overpredicts (anti-)protons by 50%!
- $p\bar{p}$ annihilation in UrQMD not strong enough to repair this
- Similar problem already seen at RHIC but not taken seriously (STAR/PHENIX disagreement)

$\bar{p}/p$ annihilation in UrQMD not strong enough to repair this
Back to the “elephant in the room”: How to eliminate the large model uncertainty in the initial eccentricity?
Two observations:

I. Shear viscosity suppresses higher flow harmonics more strongly

\[ \frac{v_n}{\varepsilon_n} \]

\[ \frac{v_3}{\varepsilon_3} \]

\[ \frac{v_4}{\varepsilon_4} \]

\[ \frac{v_5}{\varepsilon_5} \]

Alver et al., PRC82 (2010) 034913
(averaged initial conditions)

Schenke et al., arXiv:1109.6289
(event-by-event hydro)

\[ \frac{v_n(\eta/s=0.08)}{v_n(\text{ideal})} \]

\[ \frac{v_n(\eta/s=0.16)}{v_n(\text{ideal})} \]

\[ 20-30\% \]

⇒ Idea: Use simultaneous analysis of elliptic and triangular flow to constrain initial state models
(see also Bhalerao, Luzum Ollitrault, PRC 84 (2011) 034910)

II. \( \varepsilon_3 \) is \( \approx \) model independent

While \( v_4 \) and \( v_5 \) have mode-coupling contributions from \( \varepsilon_2 \), \( v_3 \) is almost pure response to \( \varepsilon_3 \) and
\( \frac{v_3}{\varepsilon_3} \approx \text{const.} \) over a wide range of centralities

⇒ Idea: Use total charged hadron \( v_{3}^{\text{ch}} \) to determine \( (\eta/s)_{\text{QGP}} \),
then check \( v_{2}^{\text{ch}} \) to distinguish between MC-KLN and MC-Glauber!
Glauber in, KLN out!
Zhi Qiu, C. Shen, UH, arXiv:1110.3033 (VISH2+1)

- Both MC-KLN with $\eta/s = 0.2$ and MC-Glauber with $\eta/s = 0.08$ give very good description of $v_2/\varepsilon_2$ at all centralities.

- Only MC-Glauber initial conditions with $\eta/s = 0.08$ describe $v_3/\varepsilon_3$

PHENIX, comparing to calculations by Alver et al. (PRC82 (2010) 034913), come to similar conclusions at RHIC energies (Adare et al., arXiv:1105.3928, and Lacey et al., arXiv:1108.0457)

- Large $v_3$ measured at RHIC and LHC requires small $(\eta/s)_{QGP} \simeq 1/(4\pi)$ unless the fluctuations predicted by both models are completely wrong and $\varepsilon_3$ is really 50% larger than we presently believe!
Conclusions

• We have come a long way over the last couple of years:
  I believe that the issue of the QGP shear viscosity at RHIC and LHC energies is now settled:

  \[
  \left( \frac{\eta}{s} \right)_{\text{QGP}}(T_c < T < 2T_c) = \frac{1}{4\pi} \pm 50\%
  \]

  A moderate increase between \(2T_c\) and \(3T_c\) can at present not be excluded but is not mandated
  by the data.

• Ingredients that matter at the 50\% level and are under control:
  – relativistic viscous fluid dynamics
  – realistic EOS with correct non-equilibrium composition in HG phase
  – microscopic description of the highly dissipative hadronic stage, including all resonance decays
  – fluctuating initial conditions, simultaneous study of \(v_2\) and \(v_3\)

• Ingredients that matter at the < 25\% level and require further study:
  – bulk viscosity
  – temperature dependence of \(\left( \frac{\eta}{s} \right)_{\text{QGP}}\)
  – pre-equilibrium flow
  – event-by-event hydro evolution vs. single-shot hydro with averaged initial profiles
  – \((3+1)\)-d vs. \((2+1)\)-d evolution
  – study of higher harmonics; influence of nucleon growth with \(\sqrt{s}\) on fluctuations
  – flow fluctuations and flow angle correlations for different harmonics

The ultimate theoretical question:

\textbf{Why is } \left( \frac{\eta}{s} \right)_{\text{QGP}} \text{ as small as it is?}
Outlook: A beautiful analogy with the Big Bang: 
The fluctuation “power spectrum” of the Little Bang


Mocsy & Sorensen, NPA855 (2011) 241, PLB705 (2011) 71
The fluctuation “power spectrum” of the Little Bang

Relating the measured “anisotropic flow power spectrum” (i.e. $v_n$ vs. $n$) to the “initial fluctuation power spectrum” (i.e. $\varepsilon_n$ vs. $n$) provides access to the QGP transport coefficients (likely not only $\eta/s$, but also $\zeta/s$, $\tau_{\pi}$, $\tau_{\Pi}$ …)

Power spectrum of initial fluctuations (in particular its $\sqrt{s}$ dependence) can (probably) be calculated from first principles via CGC effective theory (Dusling, Gelis, Venugopalan, arXiv:1106.3927)

Collisions between different species, at different collision centralities, and at different $\sqrt{s}$ create Little Bangs with characteristically different power spectra

The Concordance Model of Little Bang Cosmology!

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Supplements
Global description of AuAu@RHIC spectra and $v_2$


$$(\eta/s)_{QGP} = 0.08 \text{ for MC-Glauber and } (\eta/s)_{QGP} = 0.16 \text{ for MC-KLN works well for charged hadron, pion and proton spectra and } v_2(p_T) \text{ at all collision centralities}$$
Comparison of ALICE PbPb@LHC $v_2$ data with VISH2+1

Data: ALICE, preliminary (Snellings, Krzewicki, Quark Matter 2011)
Prediction: C. Shen et al., PRC84 (2011) 044903 (VISH2+1, MC-KLN, $\eta/s=0.2$)
In central collisions no difference between the models.

In peripheral collisions $p_T$-spectra from MC-Glauber IC too steep!

This is an artifact of single-shot hydro with averaged initial profile; for small $\eta/s = 0.08$ (but not for $\eta/s = 0.2$!), e-by-e hydro gives flatter $p_T$-spectra in peripheral collisions, due to hot spots
s95p-PCE: A realistic, lattice-QCD-based EOS

Huovinen, Petreczky, NPA 837 (2010) 26
Shen, Heinz, Huovinen, Song, PRC 82 (2010) 054904

High $T$: Lattice QCD (latest hotQCD results)

Low $T$: Chemically frozen HRG ($T_{\text{chem}} = 165$ MeV)

No softest point!
s95p-PCE: A realistic, lattice-QCD-based EOS

Huovinen, Petreczky, NPA 837 (2010) 26
Shen, Heinz, Huovinen, Song, PRC 82 (2010) 054904

Generates less radial flow than SM-EOS Q and EOS L but larger momentum anisotropy

Smooth transition leads to smaller $\delta f$ at freeze-out

$\Rightarrow$ larger $v_2$
H$_2$O: Hydro-to-OSCAR converter

Monte-Carlo interface that samples hydrodynamic Cooper-Frye spectra (including viscous correction $\delta f$) on conversion surface to generate particles at positions $x_{i}^{\mu}$ with momenta $p_{i}^{\mu}$ for subsequent propagation in UrQMD (or any other OSCAR-compatible hadron cascade afterburner)

Song, Bass, Heinz, PRC 83 (2011) 024912
VISHNU: hydro (VISH2+1) + cascade (UrQMD) hybrid

Sensitivity to $H_2O$ switching temperature:

With chemically frozen EOS (s95p-PCE), $p_T$-spectra show very little sensitivity to $T_{sw}$ (Teaney, 2000):

Song, Bass, Heinz, PRC 83 (2011) 024912

200 $A$ GeV Au+Au, $b = 7$ fm

![Graph showing $p_T$-spectra with different $T_{sw}$ values]
VISHNU: hydro (VISH2+1) + cascade (UrQMD) hybrid

Sensitivity to $H_2O$ switching temperature:

With chemically frozen EOS (s95p-PCE), $p_T$-spectra show very little sensitivity to $T_{sw}$ but $v_2$ does:

Song, Bass, Heinz, PRC 83 (2011) 024912

Viscous hydro with fixed $\eta/s = 0.08$ generates more $v_2$ below $T_c$ than does UrQMD $\implies$ UrQMD is more dissipative

VISH2+1 simulation of UrQMD dynamics requires $T$-dependent $(\eta/s)(T)$ that increases towards lower temperature
Is there a switching window in which UrQMD can be simulated by viscous hydro?

Unfortunately NO!

\((\eta/s)(T)\) extracted by trying to reproduce \(v_2\) independent of switching temperature depends on \(\delta f\) input into UrQMD from hadronizing QGP

\(\Rightarrow\) \(\delta f\) relaxes too slowly in UrQMD to be describable by viscous Israel-Stewart hydro

\(\Rightarrow\) extracted \((\eta/s)(T)\) not a proper UrQMD transport coefficient

\(\Rightarrow\) UrQMD dynamics can’t be described by viscous Israel-Stewart hydrodynamics