Study of 3D partonic picture at an EIC
Nucleon landscape

Nucleon is a many body dynamical system of quarks and gluons

Changing x we probe different aspects of nucleon wave function

How partons move and how they are distributed in space is one of the future directions of development of nuclear physics

Technically such information is encoded into Generalised Parton Distributions

See talks by Markus Deihl and Matthias Burkadt

and Transverse Momentum Dependent distributions

Plot courtesy of Christian Weiss

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Cosmic Microwave Background is the source of information on history of our universe, inflation, distribution of matter, dark matter etc.

3 Dimensional partonic picture gives us insights on the dynamics of the confined system of quarks and gluons.

It also gives information on fundamental properties of the nucleon.

Spin is one of these properties.

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Conventional inclusive processes are sensitive to longitudinal momentum fraction of hadron momenta, they give no information on spatial or momentum 3D distribution of partons.

Good knowledge of Parton Distribution Functions (PDFs) is acquired at HERA. See talk by Voica Radescu.

However, large-x behavior has still large uncertainties. Data from Jlab 12 will be important.
Our goal is to understand 3 dimensional distributions of partons. How they move, there they are located inside a nucleon.

Wigner distribution (1933) is a possibility.

\[ W(p, r) = \int d^3 \eta \, e^{i p \eta} \psi^*(r + \eta/2) \psi(r - \eta/2) \]

It gives both position and momenta.
Our goal is to understand 3 dimensional distributions of partons, How they move, there they are located inside a nucleon

Wigner (1933) distribution is a possibility

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It gives both position and momenta

Can it be measured?
Our goal is to understand 3 dimensional distributions of partons, How they move, where they are located inside of a nucleon.

Wigner (1933) distribution is a possibility

\[ W(p, r) = \int d^3\eta \ e^{ip\cdot\eta} \psi^*(r + \eta/2)\psi(r - \eta/2) \]

It gives both position and momenta

**Can it be measured?**

PROBABLY NOT!

\[ \Delta p \Delta r \geq \frac{\hbar}{2} \]

No simultaneous knowledge on position and momenta

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Wigner distribution

$W(p, r)$

Transverse Momentum Dependent distributions

$f(x, k_\perp)$

Generalized Parton Distributions

$H(x, \xi, t)$

Parton Distribution Functions

$d^2 k_\perp$

Form Factors

$dx$

$f(x)$

$F(Q^2)$

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The Wigner distribution is a fundamental concept in quantum mechanics, representing the probability distribution of finding a particle in a given state. The diagram illustrates the relationship between the Wigner distribution $W(p, r)$ and other distributions:

- **TMD (Transverse Momentum Dependent Distribution)**: $f(x, k_{\perp})$, with $d^2 k_{\perp}$.
- **GPD (Generalized Parton Distribution)**: $H(x, \xi, t)$, with $d^3 p$.

The diagram also shows the connection to other distributions:

- **PDF (Parton Distribution Function)**: $f(x)$, with $d^2 k_{\perp}$.
- **FF (Fragmentation Function)**: $F(Q^2)$, with $dx$.

The equations and diagrams are used to describe the distribution of partons within a hadron, which is crucial for understanding the structure of hadrons and their interactions. The arrows indicate the transformation or relationship between these distributions, showing how they are derived from each other under different conditions or measurements.
Transverse Momentum Dependent distributions

SIDIS

If produced hadron has low transverse momentum

$$P_{hT} \sim \Lambda_{QCD} \ll Q$$

it will be sensitive to quark transverse momentum $k_\perp$

$$l + P \rightarrow l' + h + X$$

TMD factorization

Ji, Ma, Yuan (2002)

$$\Phi_{ij}(x, k_\perp) = \int \frac{d\xi^-}{(2\pi)} \frac{d^2 \xi_\perp}{(2\pi)^2} e^{i x P^+ \xi^- - i k_\perp \xi_\perp} \langle P, S_P | \bar{\psi}_j(0) U(0, \xi) \psi_i(\xi) | P, S_P \rangle$$

Gauge Invariant

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Transverse Momentum Dependent distributions

\[ \Phi_{ij}(x, k_\perp) = \int \frac{d\xi^-}{(2\pi)} \frac{d^2\xi_\perp}{(2\pi)^2} e^{ixP^+\xi^- - ik_\perp \xi_\perp} \langle P, S_P | \bar{\psi}_j(0) \mathcal{U}(0, \xi) \psi_i(\xi) | P, S_P \rangle |_{\xi^+ = 0} \]

SIDIS in IMF:

Gauge link

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Factorization theorems

- **Related:** Factorization Theorems:
  - Semi-Inclusive deep inelastic scattering. ✓
  - Drell-Yan. ✓
  - $e^+/e^-$ annihilation. ✓
  - $p + p \rightarrow h_1 + h_2 + X$ !!

- **TMD factorization**
  \[
  \Lambda_{QCD}^2 < P_{h\perp}^2 \ll Q^2
  \]
  Sensitive to parton transverse motion.
  Ji, Ma, Yuan, Collins, Metz, Rogers, Mulders, etc

- **Collinear factorization**
  \[
  \Lambda_{QCD}^2 \ll P_{h\perp}^2, Q^2
  \]
  Sensitive to multy parton correlations.
  Qui, Sterman, Efremov, Teryaev, Kanazava, Koike, etc
TMD and Collinear factorizations

Both factorizations are consistent in the overlap region

Collins, Mulders, Ji, Qui, Yuan, Bacchetta, Kang, Boer, Koike, Vogelsang, Yuan etc

Relation of multiparton correlations and moments of TMDs

\[
\int d^2k_T \frac{k_T^2}{M} f_{1T}^+(x, k_T^2) + \text{UVCT}(\mu^2) = T_F(x, x, \mu^2) \quad f_{1T}^{+(1)} \equiv \int d^2k_T \frac{P_T^2}{2M^2} f_{1T}^+(x, k_T^2)
\]

Sivers function

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TMDs

8 functions in total (at leading Twist)

Each represents different aspects of partonic structure

Each function is to be studied

**Mulders, Tangerman (1995), Boer, Mulders (1998)**

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Let's consider unpolarised quarks inside transversely polarised nucleon

**DISTRIBUTION**

\[
f(x, k_T, S) = f_1(x, k_T^2) - \frac{[k_T \times \hat{P}] \cdot S_T}{M} f_{1T}^\perp(x, k_T^2)
\]

This one is called SIVERS function
Correlation of transverse motion and transverse spin
Sivers (1990)
Sivers function

\[ f(x, k_T, S) = f_1(x, k_T^2) - \frac{[k_T \times \hat{P}] \cdot S_T}{M} f_{1T}^{\perp}(x, k_T^2) \]

This function gives access to 3D imaging

Spin-orbit correlation

Physics of gauge links is represented

Requires Orbital Angular Momentum

EIC report, Boer, Diehl, Milner, Venugopalan, Vogelsang et al, 2011;
Access to 3D imaging

\[ f(x, k_T, S) = f_1(x, k_T^2) - \frac{[k_T \times \hat{P}] \cdot S_T}{M} f_{1T}(x, k_T^2) \]

Sivers function from experimental data HERMES and COMPASS

Anselmino et al 2005

Dipole deformation
What do we learn from 3D distributions?

\[
f(x, k_T, S_T) = f_1(x, k_T^2) - f_{1T}(x, k_T^2) \frac{k_x}{M}
\]

Suppose the spin is along Y direction: \( S_T = (0, 1) \)

Deformation in momentum space is:

\[
k_x \cdot f(k_x^2 + k_y^2)
\]

This is called “dipole” deformation.
What do we learn from 3D distributions?

\[ f(x, k_T, S_T) = f_1(x, k_T^2) - f_{1T}^\perp(x, k_T^2) \frac{k_x}{M} \]

We calculate now average shift: \[ \langle k_x \rangle \]

\[ \langle k_x \rangle = \int d^2k_T \frac{k_T^2}{2M} f_{1T}^\perp(x, k_T^2) \equiv f_{1T}^{(1)}(x) M \]

Average momentum shift is equal to the **first moment** of Sivers function
What do we learn from 3D distributions?

\[ f(x, k_T, S_T) = f_1(x, k_T^2) - f_{\perp T}(x, k_T^2) \frac{k_x}{M} \]

The same statement in figures:

No polarisation: \[ S_y \Rightarrow \langle k_x \rangle \]

Polarisation:
What do we learn from 3D distributions?

\[ f(x, k_T, S_T) = f_1(x, k_T^2) - f_{1T}^\perp(x, k_T^2) \frac{k_T1}{M} \]

The same statement in figures:

This is what we know from experimental data already:
How do we measure Sivers function?

$$A_{UT}^{\sin(\Phi_h - \Phi_S)} = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow}$$

Unpolarised electron beam

Transversely polarised proton

$$\sigma^\uparrow - \sigma^\downarrow = -f_{1T}^{\perp} \otimes d\hat{\sigma} \otimes D_{h/q} \sin(\phi_h - \phi_S)$$

$$A_{UT}^{\sin(\Phi_h - \Phi_S)} = -\frac{\sum_q e_q^2 f_{1T}^{\perp} \otimes d\hat{\sigma} \otimes D_{h/q}}{\sum_q e_q^2 f_1 \otimes d\hat{\sigma} \otimes D_{h/q}}$$

See talk by Gunar Schnell

**HERMES**

$$ep \rightarrow e\pi X, \ p_{lab} = 27.57 \ \text{GeV}.$$  

**COMPASS**

$$\mu D \rightarrow \mu\pi X, \ p_{lab} = 160 \ \text{GeV}.$$  

Anselmino et al 2010  

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What do we learn from 3D distributions?

\[ f(x, k_T, S_T) = f_1(x, k_T^2) - f_{1T}(x, k_T^2) \frac{k_T}{M} \]

The slice is at:
\[ x = 0.1 \]

Low-x and high-x region is uncertain
JLab 12 and EIC will contribute

No information on sea quarks

Picture is still quite uncertain
What do we learn from 3D distributions?

\[ f(x, k_T, S_T) = f_1(x, k_T^2) - f_{1T}(x, k_T^2) \frac{k_{T1}}{M} \]

The slice is at:

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Low-x and high-x region is uncertain
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No information on sea quarks

In future we will obtain much clearer picture
Colored objects are surrounded by gluons, profound consequence of gauge invariance. Sivers function has opposite sign when gluon couple after quark scatters (SIDIS) or before quark annihilates (Drell-Yan)

\[ f_{1T}^{\perp SIDIS} = - f_{1T}^{\perp DY} \]

One of the main goals is to verify this relation. It goes beyond “just” check of TMD factorization. Motivates Drell-Yan experiments

AnDY, COMPASS, JPARC, PAX etc
Barone et al., Anselmino et al., Yuan, Vogelsang, Schlegel et al., Kang, Qiu, Metz, Zhou

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TMD theoretical challenges

• Evolution and soft gluon resummation
• Global study at Next-to-Leading order
• Relation to Orbital Angular Momentum

Many more other questions

• What is the \( k_t \) distributions of partons – gaussian, powerlike, sign changing?
• What is the difference of \( k_t \) distributions of quarks and sea quarks?
• How to explore higher twist TMDs?
• How to explore distribution and fragmentation TMDs in a satisfactory way?
• etc
Collins-Soper-Sterman factorization can be used

\[
\frac{\partial \ln \tilde{F}(x, b_\perp, \mu, \zeta)}{\partial \ln \zeta} = \tilde{K}(b_\perp, \mu)
\]

\[
\frac{d\tilde{K}(b_\perp, \mu)}{d \ln \mu} = -\gamma K(\mu)
\]

\[
\frac{d\tilde{F}(x, b_\perp, \mu, \zeta)}{d \ln \mu} = \gamma F(\mu, \zeta)
\]

CS kernel in coordinate space

TMDs change with energy and resolution scale

 Relevant to EIC

TMD:
Collins 2011
Rogers, Aybat 2011
Twist-3:
Kang, Xiao, Yuan 2011
Koike, Vogelsang 2011

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Can we see signs of evolution in the experimental data?

**TMD evolution**

COMPASS data is at
\[ \langle Q^2 \rangle \simeq 3.6 \ (GeV^2) \]

HERMES data is at
\[ \langle Q^2 \rangle \simeq 2.4 \ (GeV^2) \]
Can we explain the experimental data?
Convention method is to apply DGLAP evolution only

Aybat, AP, Rogers 2011

COMPASS dashed line
\[ \langle Q^2 \rangle \simeq 3.6 \ (GeV^2) \]

HERMES solid line
\[ \langle Q^2 \rangle \simeq 2.4 \ (GeV^2) \]
Can we explain the experimental data? Full TMD evolution is needed!

Aybat, AP, Rogers 2011

COMPASS dashed line

\[ \langle Q^2 \rangle \simeq 3.6 \ (GeV^2) \]

HERMESIS solid line

\[ \langle Q^2 \rangle \simeq 2.4 \ (GeV^2) \]
TMD evolution

This is the first implementation of TMD evolution for observables

Asymmetry changes with $Q^2$

Functions change with energy

Phenomenological analysis with evolution is now possible

Aybat, AP, Rogers 2011

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Kinematics

\[ Q^2 \sim sxy \]

Jlab 12 and future Electron Ion Collider are complimentary.

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Jlab 12 and future Electron Ion Collider are complimentary.

Jlab and EIC are going to provide fine 4D binning of the data. Exact knowledge of evolution is crucial.
Future improvement

What do we expect at JLab?

One example
TMD from Jlab future data:
JLab 12 on 3HE target.

Very big improvement in terms of our knowledge

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Future improvement

What do we expect at EIC?

One example
TMD from EIC
future data

Very big improvement
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Possible relations of TMDs and OAM

Bacchetta, Radici 2011 argue that

\[ f_{1T}^{\perp (0)}(x) \simeq I(x)E(x, 0, 0) \]

Inspired by model Relations, not full QCD

So called “lensing” function

Burkardt, Metz, etc

Making direct connection to total OAM from Ji's sum rule:

\[
J_q = \frac{1}{2} \int_0^1 dx x (H_q(x, 0, 0) + E_q(x, 0, 0))
\]

\[
J_u = 0.266, \quad J_d = -0.012 \quad \text{at} \quad Q^2 = 1(\text{GeV}^2)
\]

Asumptions based on model calculations of course, but might be interesting.

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Gluons distributions

- Gluon TMDs
- Linearly polarized gluons can be accessed in various channels
- Opportunities of studies at EIC, LHeC
  - $h_{1\perp}^g$ may contribute to Higgs production, resolve its parity
    - Boer, Brodsky, Mulders, Pisano 2011
    - Qiu, Vogelsang, Schlegel 2011
    - Boer, den Dunnen, Pisano, Schlegel, Vogelsang 2011
- Tri-gluon correlations, Qiu-Sterman matrix elements, complete classification
  - Koike, Tanaka
- TMD SSA in open charm
  - Godbole, Misra, Mukherjee, Rawoot 2011

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TMD experimental challenges

• 4D binning of observables $x, z, Q^2, P_{h\perp}$

• Different targets: proton, neutron, deuteron

• Different final state hadrons $\pi, K$
  open charm

All this helps to do correct flavour decomposition and correct analysis.
CONCLUSIONS

• Studies of 3D distributions represent big part of future of nuclear physics

• EIC is an ideal place to explore GPDs and TMDs

• Theory and phenomenology have made a lot of progress in recent years

• We are going to see more progress in future
CONCLUSIONS

• Studies of 3D distributions represent big part of future of nuclear physics

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• Theory and phenomenology have made a lot of progress in recent years

• We are going to see more progress in future

• We are looking forward to EIC!
QCD EVOLUTION 2012

http://www.jlab.org/conferences/qcd2012/

May 14 - 17, 2012
Jefferson Lab
Newport News, Virginia, USA

Organizing committee:
Alexei Prokudin, Chair
Anatoly Radyushkin
Ian Balitsky
Leonard Gamberg
Harut Avakian

Alexei Prokudin
HUGS 2012

http://www.jlab.org/hugs/

Summer school: 27th Annual Hampton University Graduate Studies Program. Covers theoretical and experimental aspects of nuclear physics.

Jefferson Lab, Newport News, Virginia
June 4 - June 22, 2012

Fellowships are available and will cover tuition, fees, room and board

The deadline for application submittal is April 2, 2012

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